# Coming to Grips with $\mathbf{N}-\mathbf{H} \cdots \mathbf{N}$ Bonds. 1. Distance Relationships and Electron Density at the Bond Critical Point 

Osvald Knop,* Kathryn N. Rankin, and Russell J. Boyd ${ }^{\dagger}$<br>Department of Chemistry, Dalhousie University, Halifax NS, Canada B3H 4J3

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#### Abstract

In an attempt to discover and analyze trends in distance relationships and properties at the bond critical point (BCP) in linear or near-linear $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ hydrogen bonds, the geometry of such bonds in a large number of suitable simple chemical species was optimized at the RHF/6-31G** level. The results for 67 of these are reported here; the geometry of 19 of them was optimized also at the MP2/6-31G** level. Correlations between the internuclear $\mathrm{N}-\mathrm{H}, \mathrm{H} \cdots \mathrm{N}$, and $\mathrm{N} \cdots \mathrm{N}$ separations as well as between the $\mathrm{N} \cdots \mathrm{BCP}$ and $\mathrm{H} \cdots \mathrm{BCP}$ distances for these data sets and for different model functions are described in detail. The special case of symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds is discussed; comparison with available experimental evidence shows that the correlation functions derived from the ab initio data have useful predictive value for crystallographic determinations involving short $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds. Analysis of the correlation between the $d(\mathrm{~N}-\mathrm{H})$ distance and the electron density $\rho_{\mathrm{c}}$ at the BCP has shown that although acceptable $d, \rho_{\mathrm{c}}$ representations are obtained when $\rho_{\mathrm{c}}$ is fitted over the entire $d(\mathrm{~N}-\mathrm{H})$ range by a single model function, significantly better fits result for both the $6-31 \mathrm{G}^{* *}$ and the MP2/6-31G** set when two separate regression functions of the same type are used, one for the covalent and another for the $\mathrm{H} \cdots \mathrm{N}$ bonds. The implications of these findings are discussed. The results of the correlation analysis of the curvatures $\lambda_{\mathrm{i}}$, the Laplacians $\nabla^{2} \rho_{\mathrm{c}}$, and the kinetic energy densities at the BCP, based on the data presented in this paper, will be reported in a subsequent paper, together with some aspects of the energy of formation of the $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bonds.


## Prefatory Note

The RHF/6-31G** geometry optimization used in most of the calculations in this paper was adopted mainly to generate, within the intended scope of this exploratory study, a homogeneous body of results that would be sufficiently large to reveal and describe the existence of such consistent trends in the properties of $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds as may be evident in this model. This optimization was manageable at the practical level, and it rendered equilibrium $\mathrm{N}-\mathrm{H}-\mathrm{N}$ geometries that are comparable to experiment. It is expected that the analysis and conclusions presented here will provide a basis for further exploration and verification, at a more elaborate theory level and by experiment, of the trends we report. The present paper is concerned with those properties in Tables 1-4 that relate to the geometry of, and the electron density at the bond critical points in, the $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds investigated. The remaining data in Tables $1-4$ will be analyzed for correlations in a subsequent paper, now in preparation.

## Notation

In a complete $N_{d}-H \cdots N_{a}$ hydrogen bond, $N_{d}$ is the donor and $\mathrm{N}_{\mathrm{a}}$ the acceptor atom. Unprimed quantities (e.g., the internuclear distance $d$, the electron density $\rho_{\mathrm{c}}$ at the bond critical point BCP, the Laplacian $\nabla^{2} \rho_{c}$ ) refer indiscriminately to the covalent $\mathrm{N}_{\mathrm{d}}-\mathrm{H}$ and the H -bond $\mathrm{H} \cdots \mathrm{N}_{\mathrm{a}}$ components of the bond. Primed entities (e.g., $d^{\prime}$, the bond critical point $\mathrm{X}^{\prime}$ ) refer to the $\mathrm{N}_{\mathrm{d}}-\mathrm{H}$ component and the doubly primed entities (e.g., $d^{\prime \prime}, \mathrm{X}^{\prime \prime}$ )

[^0]to the $\mathrm{H} \cdots \mathrm{N}_{\mathrm{a}}$ component. Other symbols are defined in Tables 1 and 2 and as they are encountered in the text for the first time. Numerical values of the quantities discussed are given in atomic units (au) except for the distances, which are given in $\AA$ for convenience of comparison with experimental values. Throughout, HF will refer to complete RHF/6-31G** geometry optimizations unless stated otherwise. Together, the species listed in Table 1 constitute the sample $\mathbf{S}$.

## Introduction

Although considerably less attention has been lavished on an understanding of the properties of $\mathrm{N}-\mathrm{H}-\mathrm{N}$ hydrogen bonds than on their oxygen-involving heteronuclear counterparts, ${ }^{1}$ the $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds do not lack in importance and interest, not least because of their potential involvement in biological systems. Steiner ${ }^{2}$ surveyed the geometry of $31 \mathrm{~N}-\mathrm{H}-\mathrm{N}$ bonds ( $1.6 \AA$ $<d^{\prime \prime}<2.4 \AA$ ) in which the H atom had been located by highquality neutron diffraction. He found a high degree of correlation between the $d^{\prime}$ and $d^{\prime \prime}$ internuclear distances. This he was able successfully to represent by the function

$$
\begin{equation*}
d^{\prime}=d_{0}-b \ln \left\{1-\exp \left[\left(d_{0}-d^{\prime \prime}\right) / b\right]\right\} \tag{1}
\end{equation*}
$$

which is based on an adaptation of Pauling's bond order equation ${ }^{3}, D(n)=D(1)-B \ln n$, to conjugate bond lengths in 3c4e bonds: $d^{\prime}\left(s^{\prime}\right)=d_{0}-b \ln s^{\prime}, d^{\prime \prime}\left(s^{\prime \prime}\right)=d_{0}-b \ln s^{\prime \prime}, s^{\prime}+$ $s^{\prime \prime}=1\left(s^{\prime}, s^{\prime \prime}=\right.$ bond orders, $d_{0}=$ limiting $\mathrm{N}-\mathrm{H}$ bond length $) .{ }^{4}$ The function is symmetric about the $d^{\prime}=d^{\prime \prime}$ line, i.e., $d^{\prime}$ and $d^{\prime \prime}$ are interchangeable. In a symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bond, $d^{\prime}=d^{\prime \prime}$ $=d_{\mathrm{sym}}=d_{0}+b \ln 2$. For Steiner's sample, the regression
coefficients were

$$
\begin{equation*}
d_{0}=0.996 \AA, b=0.381 \AA \tag{2}
\end{equation*}
$$

yielding $d_{\text {sym }}=1.260 \AA$. This value is in agreement with the $d_{\text {sym }}$ distance reported ${ }^{5}$ for the shortest known, linear symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bond, that in the inside-protonated (intrabridgehead) cation of 1,6-diazabicyclo[4.4.4]tetradecane (henceforth referred to as $[4.4 .4]^{+}$). The $\mathrm{N} \cdots \mathrm{N}$ distance in the chloride of this cation (X-ray diffraction at room temperature) is quoted as $2.526(3)$ $\AA$; the centrosymmetric (single-well) geometry of this bond has been deduced from the X-ray crystallographic $D_{3}$ symmetry of the cation and the thermal parameters of the H atom, from neutron diffraction ${ }^{6,7}$ at 20 K (not included in Steiner's sample), ${ }^{8}$ and from NMR and IR spectra. ${ }^{9}$ The $d_{\text {sym }}=1.26 \AA$ is also compatible with the ab initio estimates obtained by Scheiner ${ }^{10}$ from his analysis of the requirements for the existence of symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds.

For linear or near-linear $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bonds, (1) can be reformulated to include the $\mathrm{N} \cdots \mathrm{N}$ distance $D=d^{\prime}+d^{\prime \prime}$ :

$$
\begin{array}{r}
D=d_{0}+d^{\prime \prime}-b \ln \left\{1-\exp \left[\left(d_{0}-d^{\prime \prime}\right) / b\right]\right\}=d_{0}+d^{\prime}- \\
b \ln \left\{1-\exp \left[\left(d_{0}-d^{\prime}\right) / b\right]\right\} \tag{3}
\end{array}
$$

This form is particularly useful in that it permits estimates of the position of the H atom in such bonds from the positions of the two N atoms, which in structures determined from X-ray data are more accurately known than the position of the H atom (see, for example, ref 11).

The rapidly expanding activity and interest (including our own ${ }^{12}$ ) in experimental determinations of charge densities and bond critical parameters (the electron density $\rho_{\mathrm{c}}$ and the Laplacian $\nabla^{2}\left(\rho_{\mathrm{c}}\right)$ at the BCP, etc.) of hydrogen bonds ${ }^{13}$ have prompted us to look for an account of the properties of $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bonds that would combine bond geometry with these bond critical parameters and result in a predictive scheme. Basing such an account on the experimental results available to date has its difficulties. In Steiner's sample, the shortest bonds have $d^{\prime \prime}$ not below $1.6 \AA$, and with the exception of $[4.4 .4]^{+}$, even in the bonds reported as symmetric the $d^{\prime \prime}$ is not below $\sim 1.3 \AA$ (see below). We do not know whether this gap in the $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ geometry exists because of an as yet insufficient experimental effort (preparability, crystallography, MW spectroscopy) or for other reasons, possibly intrinsic. Second, bond critical parameters determined by experiment have so far been reported for only a few $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bonds. In some of them, the H atom has not been located by matching neutron diffraction, and the haphazard accumulation of experimental determinations does not ensure a uniform distribution of $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ geometries over the $d$ range. Under these circumstances, recourse to a reasonably extensive, systematic ab initio study seemed to be the most promising approach to elucidating properties of $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bonds as a class. This paper is an account of such a goal-oriented exploratory investigation.

Effort was made for the sample $\mathbf{S}$ (Table 1) to be representative, both as to geometry and chemical variety, over as wide a $d$ range as possible. This explains the inclusion in $\mathbf{S}$ of a number of exotic and hypothetical molecular species, some of which may be inaccessible to experiment, being incapable, under laboratory conditions, of independent existence vis-à-vis more stable competing species or systems. The well-tested and documented RHF/6-31G** scheme was employed to generate optimized molecular geometries and bond critical parameters sufficiently realistic to reveal underlying trends and sufficiently accurate for such trends to be evaluated analytically or statisti-
cally. This scheme was used throughout except for occasional MP2/6-31G** level verification checks, as described.

The optimizations were performed using GAUSSIAN94 ${ }^{14}$ and supplemented using the AIMPAC ${ }^{15}$ program package. The results listed in Tables 1 and 2 are those for point-group symmetries that yielded the lowest total electronic energies $E$. The highest geometrically possible symmetry was assumed for the initial geometry and the result was compared to subsequent optimizations in one or more subgroup symmetries, as required (cf. Table 1). For a few of the species $(\mathbf{1 7}, \mathbf{5 9}, \mathbf{6 4}, \mathbf{6 5})$, acceptable optimization was not achieved in any of the symmetries tried (one or more imaginary frequencies, gross discrepancies in parameter correlations). However, these species subsequently were successfully optimized in MP2/6-31G**, which confirmed their ab initio existence as H -bonded species. The results of the MP2 optimizations are collected in Tables 3 and 4.

Details of the optimized equilibrium molecular geometries (Cartesian coordinates of the atoms, bond lengths and angles other than those in the $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds, etc.) were thought too voluminous even for the Supporting Information. However, structural and other features of particular interest will be extracted for separate publication elsewhere.

## Distance Relationships

Relationships between $\boldsymbol{d}^{\prime}, \boldsymbol{d}^{\prime \prime}$, and $\boldsymbol{D}$. The $d^{\prime}, d^{\prime \prime}$ pairs in $\mathbf{S}$ extend the $d^{\prime \prime}$ range of Steiner's sample (Figure 1A). Like those of the Steiner set, they are all well fitted by (1) as the model function. The regression of $d^{\prime}$ on $d^{\prime \prime}$ yields

$$
\begin{array}{r}
d_{0}=0.981(3) \AA, b=0.386(9) \AA, d_{\mathrm{sym}}=1.249(9) \AA \\
D_{\min }=2.497(19) \AA \tag{4}
\end{array}
$$

$\left(r^{2}=0.896, \sigma=0.011 \AA\right)$; the regression coefficients $d_{0}$ and $b$ are strongly correlated, $r=-0.93$ ). The residuals are distributed quite uniformly. The scatter in Figure 1A may thus be attributed to the natural variation in $\mathbf{S}$.

Although the regression coefficients (4) are similar to those of Steiner's regression (2) for the experimental data set, statistical tests on the sums of squares of the residuals show that representing the $\mathbf{S}$ data by (1) + (2) can be strongly rejected in favor of the independently fitted (1) $+(4)$ function. The difference between Steiner's $d^{\prime}[(1)+(2)]$ function, for the experimental data set, and our $d^{\prime}[(1)+(4)]$, for the HF set, is everywhere positive. It increases with decreasing $d^{\prime \prime}$ up to $\sim 0.017 \AA$ at the short end of our $d^{\prime \prime}$ range and is $\sim 0.023 \AA$ at $d_{\text {sym }}$; with increasing $d^{\prime \prime}$ it tends to a residual asymptotic value of $\sim 0.015 \AA$. The optimized $d^{\prime}, d^{\prime \prime}$ distances are thus consistently undervalued relative to experiment (at finite temperatures), in agreement with the general expectation for $6-31 G^{*} *_{-}$ optimized internuclear distances. To this extent, the optimized $d^{\prime}, d^{\prime \prime}$ are validated by experiment to a remarkably high degree over the entire $d^{\prime \prime}$ range examined. The difference between Steiner's $d_{\text {sym }}=1.260 \AA$ and that from (1) $+(4)$ is $0.012 \AA$ ( $0.017 \AA$ if taken along the $45^{\circ}$ symmetry line of the plot), i.e., within $2 \sigma$ of $d_{\text {sym }}$ from (4).

It is to be noted that while the $d^{\prime \prime}$ values of Table 2 extend quite far, to $\sim 2.5 \AA$ and in a coherent sequence of optimized geometries, the smallest $d^{\prime \prime}($ in $\mathbf{1})$ is still $\sim 0.22 \AA$ away from the $d_{\text {sym }}$ estimated from the regression. This is elaborated upon below.

The internuclear distance relationships can be presented in other instructive ways. For linear $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bonds the means of the conjugate $d^{\prime}, d^{\prime \prime}$ pairs, when plotted against $D$ (Figure $2 \mathrm{~A})$, would fall by definition on a rectilinear locus $\left(d^{\prime}+d^{\prime \prime}\right) / 2$

TABLE 1: RHF/6-31G*** Optimized Equilibrium Molecular Geometries ${ }^{a}$

| configuration $^{\text {b }}$ | $\mathrm{PG}^{c}$ | $\mathrm{N}_{\mathrm{d}} \mathrm{HN}_{\mathrm{a}}{ }^{\text {d }}$ (deg) | -E | $-E_{\mathrm{f}}{ }^{e}$ | $\epsilon\left(\mathrm{N}_{\mathrm{d}}\right)$ | $\epsilon(\mathrm{H})$ | $\epsilon\left(\mathrm{N}_{\mathrm{a}}\right)$ | other PGs tried ${ }^{f}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1\left[\mathrm{~F}_{3} \mathrm{NH} \cdots \mathrm{NCF}\right]^{+}$ | $C_{3 v}$ | (180) | 544.51968 | 0.05101 | 0.846 | 0.558 | -0.502 | $C_{1}$ |
| $2[\mathrm{FCNH} \cdots \mathrm{NCF}]^{+}$ | $C_{\infty}$ | (180) | 383.71465 | 0.04022 | -0.389 | 0.559 | -0.472 | [ $C_{\text {S }}$ ], $C_{1}$ |
| 3 [ $\mathrm{HCNH} \cdots \cdot \mathrm{NCH}]^{+}$ | $C_{\infty}$ | (180) | 186.08449 | 0.04205 | -0.417 | 0.546 | -0.488 | [ $D_{\infty}$ ],$C_{\text {s }}$ |
| $4[\mathrm{LiCNH} \cdots \mathrm{NCLi}]^{+}$ | $C_{\infty}$ | (180) | 199.97935 | 0.05528 | -0.482 | 0.511 | -0.545 | $C_{\text {s }}$ |
| $5[\mathrm{OCNH} \cdot \cdots \mathrm{NCO}]^{-}$ | $C_{\text {s }}$ | 179.6 | 335.00171 | 0.04025 | -0.669 | 0.473 | -0.736 | [ $\left.D_{\infty}\right]$ ], [ $\left.C_{\infty \nu v}\right],\left[C_{2 h}\right],\left[C_{2 v}\right]$ |
| $6[\mathrm{HCNH} \cdots \mathrm{NCF}]^{+}$ | $C_{\infty}$ | (180) | 284.90120 | 0.03863 | -0.411 | 0.539 | -0.471 | $C_{s}$ |
| $7[\mathrm{SCNH} \cdots \mathrm{NCS}]^{-}$ | $C_{\infty}$ | (180) | 980.31416 | 0.04569 | -0.537 | 0.482 | -0.603 | [ $D_{\infty}$ ], $C_{s}$ |
| $8\left[\mathrm{FH}_{2} \mathrm{NH} \cdots \mathrm{NH}_{3}\right]^{+}$ | $C_{s}$ | 174.6 | 211.52114 | 0.05078 | -0.189 | 0.494 | -0.878 |  |
| $9\left[\mathrm{~F}_{3} \mathrm{NH} \cdots \mathrm{NCCN}\right]^{+}$ | $C_{3 v}$ | (180) | 537.39817 | 0.03555 | 0.870 | 0.534 | -0.584 | $C_{1}$ |
| $10[\mathrm{NCCNH} \cdots \mathrm{NCCN}]^{+}$ | $C_{\infty}$ | (180) | 369.47087 | 0.02843 | -0.491 | 0.534 | -0.550 | [ $D_{\infty h}$ ], $C_{s,}, C_{1}$ |
| $11[\mathrm{CNH} \cdots \mathrm{NC}]^{-}$ | $C_{\infty}$ | (180) | 185.19336 | 0.04799 | -0.510 | 0.465 | -0.616 | [ $D_{\infty}$ ], $C_{s}$ |
| $12\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NCS} \cdots \mathrm{HNH}_{3}\right]^{+}$ | $C_{s}$ | 176.4 | 603.19418 | 0.22919 | -0.718 | 0.482 | -0.606 |  |
| $\mathbf{1 3}[\mathrm{CNH} \cdots \mathrm{NCO}]^{-}$ | $C_{\infty}$ | (180) | 260.10042 | 0.04864 | -0.504 | 0.466 | -0.739 | $C_{s}$ |
| $14\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NCLi}\right]^{+}$ | $C_{3 v}$ | (180) | 156.38789 | 0.06986 | -0.718 | 0.474 | -0.574 | $C_{s,} C_{1}$ |
| 15 [FCNH $\cdots$ NCCN] ${ }^{+}$ | $C_{\infty}$ | (180) | 376.59529 | 0.02691 | -0.373 | 0.531 | -0.542 | $C_{1}$ |
| $16[\mathrm{HCNH} \cdots \mathrm{NCCN}]^{+}$ | $C_{\infty}$ | (180) | 277.78232 | 0.02580 | -0.401 | 0.517 | -0.537 | $C_{s}$ |
| 17 [CNH $\cdots$ NNN] ${ }^{-}$ | $C_{s}$ | 176.8 | 256.16140 | 0.04068 | -0.501 | 0.449 | -0.710 | [ $C_{\infty}$ ] $],\left[C_{1}\right]$ |
| 18 [OCNH $\cdots$ NNN] ${ }^{-}$ | $C_{1}$ | 175.8 | 331.06322 | 0.03610 | -0.701 | 0.452 | -0.711 | $\left[C_{s}\right.$ ] |
| $19\left[\mathrm{~F}_{2} \mathrm{HNH} \cdots \mathrm{NCCN}^{+}\right.$ | $C_{s}$ | 178.3 | 438.63903 | 0.02853 | 0.390 | 0.489 | -0.559 |  |
| $20 .\left[\mathrm{FH}_{2} \mathrm{NH} \cdots \mathrm{NH}_{2} \mathrm{~F}\right]^{+}$ | $C_{s}$ | 144.1 | 310.27404 | 0.03631 | -0.112 | 0.444 | -0.307 |  |
| $21\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NH}_{3}\right]^{+}$ | $C_{3 v}$ | (180) | 112.78284 | 0.04177 | -0.712 | 0.459 | -0.878 | [ $D_{3 d}$ ], $C_{S,} C_{1}$ |
| $22[\mathrm{CNH} \cdots \mathrm{NCS}]^{-}$ | $C_{\infty}$ | (180) | 582.76701 | 0.03507 | -0.498 | 0.444 | -0.591 | $C_{s}$ |
| $23\left[\mathrm{~F}_{2} \mathrm{HNH} \cdots \mathrm{NHF}_{2}\right]^{+}$ | $C_{s}$ | 172.4 | 507.78940 | 0.03137 | 0.392 | 0.490 | 0.212 |  |
| $24\left[\mathrm{LiCNH} \cdots \mathrm{NH}_{2} \mathrm{~F}\right]^{+}$ | $C_{1}$ | 172.5 | 255.13858 | 0.02408 | -0.481 | 0.474 | -0.310 |  |
| 25 [OCNH $\cdots$ NCS] ${ }^{-}$ | $C_{s}$ | 177.4 | 657.66839 | 0.02843 | -0.833 | 0.543 | -0.819 | $\left[C_{\infty \nu}\right]$ |
| $26 \mathrm{SCNH} \cdots \mathrm{NCLi}$ | $C_{s}$ | 179.9 | 590.19244 | 0.02380 | -0.558 | 0.431 | -0.491 |  |
| 27 [LiCNH $\cdots \mathrm{NCH}]^{+}$ | $C_{s}$ | 174.7 | 193.05438 | 0.02567 | -0.470 | 0.462 | -0.455 | [ $C_{\infty}$ ] $],\left[C_{1}\right]$ |
| 28 SCNH $\cdots \mathrm{NH}_{3}$ | $C_{3 v}$ | (180) | 546.61156 | 0.01987 | -0.588 | 0.438 | -0.843 | $C_{s}$ |
| $29[\mathrm{LiCNH} \cdot \cdots \mathrm{NCF}]^{+}$ | $C_{\infty}$ | (180) | 291.87255 | 0.02371 | -0.471 | 0.462 | -0.443 | $C_{s}$ |
| 30 [NNNH $\cdots$ NNN] ${ }^{-}$ | $C_{s}$ | 179.5 | 327.13346 | 0.02946 | -0.559 | 0.410 | -0.701 | [ $\left.D_{\infty}\right]$ ], [ $\left.C_{\infty \nu v}\right],\left[C_{2 h}\right],\left[C_{2 v}\right]$ |
| $31\left[\mathrm{FH}_{2} \mathrm{NH} \cdots \mathrm{NCCN}\right]^{+}$ | $C_{s}$ | 179.6 | 339.88937 | 0.02335 | -0.133 | 0.463 | -0.539 |  |
| $32\left[\mathrm{~F}_{3} \mathrm{NH} \cdots \mathrm{NF}_{3}\right]^{+}$ | $C_{3 v}$ | (180) | 705.32217 | 0.01071 | 0.909 | 0.485 | 0.756 | $C_{1}$ |
| $33\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NCH}\right]^{+}$ | $C_{3 v}$ | (180) | 149.45569 | 0.03302 | -0.692 | 0.450 | -0.488 | $C_{s}$ |
| $34\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NCF}\right]^{+}$ | $C_{3 v}$ | (180) | 248.27300 | 0.03020 | -0.692 | 0.451 | -0.478 | $C_{s}$ |
| $35 \mathrm{CNH} \cdots \mathrm{NCLi}$ | $C_{\infty}$ | (180) | 192.65165 | 0.01954 | -0.507 | 0.416 | -0.483 | $C_{s}$ |
| $36 \mathrm{CNH} \cdots \mathrm{NH}_{3}$ | $C_{3 v}$ | (180) | 149.07218 | 0.01702 | -0.525 | 0.424 | -0.834 | $C_{s}$ |
| $37\left[\mathrm{HCNH} \cdots \mathrm{NF}_{3}\right]^{+}$ | $C_{3 v}$ | (180) | 445.71275 | 0.00739 | -0.400 | 0.478 | -0.776 | $C_{s}$ |
| 38 [LiCNH $\cdots$ NCCN] ${ }^{+}$ | $C_{\infty}$ | (180) | 284.75761 | 0.01483 | -0.472 | 0.452 | -0.485 | $C_{s}$ |
| 39 [NNNH $\cdots \cdot \mathrm{NCS}^{-}$ | $C_{s}$ | 171.5 | 653.74066 | 0.02383 | -0.556 | 0.405 | -0.601 |  |
| $40\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NCCN}\right]^{+}$ | $C_{3 v}$ | (180) | 241.15590 | 0.01915 | -0.687 | 0.443 | -0.521 | $C_{s}$ |
| $41\left[\mathrm{LiCNH} \cdots \mathrm{NHF}_{2}{ }^{+}\right.$ | $C_{s}$ | 172.6 | 353.91083 | 0.02510 | -0.478 | 0.452 | 0.246 |  |
| $42{ }^{\text {f }} \mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NCS}$ | $C_{s}$ | 141.3 | 546.58739 | 0.16793 | -0.672 | 0.405 | -0.563 |  |
| 43 OCNH $\cdots$ NCLi | $C_{s}$ | 176.0 | 267.55428 | 0.01575 | -0.707 | 0.407 | -0.488 | $\left[C_{\infty}{ }^{\text {c }}\right.$ ] |
| $44 \mathrm{OCNH} \cdots \mathrm{NH}_{3}$ | $C_{s}$ | 177.1 | 223.97519 | 0.01362 | -0.716 | 0.411 | -0.830 | $\left[C_{3 v}\right]$ |
| 45 SCNH $\cdots \mathrm{NCH}$ | $C_{s}$ | 179.3 | 583.28459 | 0.01131 | -0.577 | 0.412 | -0.413 |  |
| 46 SCNH $\cdots$ NCF | $C_{s}$ | 179.6 | 682.10392 | 0.01051 | -0.578 | 0.411 | -0.403 |  |
| $47 \mathrm{CNH} \cdots \mathrm{NCH}$ | $C_{\infty}$ | (180) | 185.74654 | 0.00979 | -0.511 | 0.401 | -0.409 | $C_{s}$ |
| $48 \mathrm{CNH} \cdots \mathrm{NCF}$ | $C_{\infty}$ | (180) | 284.56605 | 0.00917 | -0.511 | 0.400 | -0.400 | $C_{s}$ |
| $49 \mathrm{NNNH} \cdot \cdots \mathrm{NCLi}$ | $C_{s}$ | 164.9 | 263.62872 | 0.01333 | -0.539 | 0.373 | -0.490 |  |
| 50 SCNH $\cdots$ NCCN | $C_{s}$ | 179.8 | 674.99372 | 0.00636 | -0.587 | 0.402 | -0.432 |  |
| 51 NNNH $\cdots \mathrm{NH}_{3}$ | $C_{s}$ | 180.0 | 220.04918 | 0.01074 | -0.542 | 0.375 | -0.819 | $\left[C_{3 v}\right], C_{1}$ |
| $52 \mathrm{SCNH} \cdots \mathrm{NHF}_{2}$ | $C_{s}$ | 171.6 | 744.14831 | 0.01342 | -0.606 | 0.403 | 0.281 |  |
| 53 OCNH $\cdots$ NCH | $C_{s}$ | 164.9 | 260.65038 | 0.00722 | -0.704 | 0.384 | -0.413 | $\left[C_{\infty}{ }^{\text {] }}\right.$ ] |
| 54 CNH $\cdots$ NCCN | $C_{\infty}$ | (180) | 277.45658 | 0.00575 | -0.512 | 0.392 | -0.425 | $C_{s}$ |
| 55 OCNH $\cdots$ NCF | $C_{s}$ | 177.9 | 359.47045 | 0.00716 | -0.704 | 0.384 | -0.403 |  |
| $56\left[\mathrm{LiCNH} \cdots \mathrm{NF}_{3}\right]$ | $C_{3 v}$ | (180) | 452.69520 | 0.00357 | -0.475 | 0.423 | -0.807 | $C_{s}$ |
| 57 OCNH $\cdots$ NCCN | $C_{s}$ | 174.9 | 352.36153 | 0.00429 | -0.701 | 0.374 | -0.423 |  |
| 58 NNNH $\cdots$. ${ }^{\text {NCH }}$ | $C_{s}$ | 171.5 | 256.72625 | 0.00621 | -0.526 | 0.355 | -0.412 |  |
| $59\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NF}_{3}\right]^{+}$ | [ $C_{3 v}$ ] | (180) | 409.08959 | [0.00400] | -0.673 | 0.415 | 0.794 | [ $C_{1}$ ] |
| 60 NNNH $\cdot \cdots$ NCF | $C_{s}$ | 168.1 | 355.54598 | 0.00581 | -0.525 | 0.352 | -0.406 |  |
| 61 NNNH $\cdots$ NCCN | $C_{s}$ | 174.3 | 348.43751 | 0.00340 | -0.519 | 0.343 | -0.419 |  |
| $62 \mathrm{SCNH} \cdots \mathrm{NF}_{3}$ | $C_{s}$ | 178.0 | 842.93768 | 0.00148 | -0.618 | 0.374 | 0.835 |  |
| $63 \mathrm{CNH} \cdots \mathrm{NF}_{3}$ | $C_{3 v}$ | (180) | 445.40115 | 0.00148 | -0.511 | 0.366 | 0.837 | $C_{s}$ |
| $64 \mathrm{OCNH} \cdots \mathrm{NF}_{3}$ | [ $C_{\text {s }}$ ] | 175.4 | 520.30714 | [0.00106] | -0.694 | 0.347 | 0.841 | [ $C_{3 v}$ ], [ $C_{1}$ ] |
| 65 NNNH $\cdots \mathrm{NF}_{3}$ | $\left[C_{s}\right]$ | 172.7 | 516.38374 | [0.00078] | -0.506 | 0.318 | 0.843 | $\left[C_{3 v}\right],\left[C_{s, s y n}\right],\left[C_{1}\right]$ |
| $66 \mathrm{H}_{3} \mathrm{NH}^{+}$ | $T_{d}$ |  | 56.54553 |  | -0.655 | 0.414 |  |  |
| $67 \mathrm{~F}_{3} \mathrm{NH}^{+}$ | $C_{3 v}$ |  | 352.77140 |  | 0.964 | 0.458 |  |  |

[^1]TABLE 2: Internuclear and Atom $\cdots$ BCP Distances ( $(\AA)$, Electron Densities $\rho_{\mathrm{c}}(r)$, Laplacians $\nabla^{2} \rho_{\mathrm{c}}(r)$, Eigenvalues $\lambda_{\mathrm{i}}$ of the Laplacians, Ellipticities $\eta$, and Kinetic Energy Densities $G(r)$ and $K(r)$ at the Bond-Critical Points (all in au) in the 6-31G $\mathbf{G}^{* *}$-Optimized Species of Table 1 (Arranged in the Order of Increasing $\left.\boldsymbol{D}\right)^{a}$

|  | D | $d^{\prime} ; d^{\prime \prime}$ | $x^{\prime} ; x^{\prime \prime}$ | $\rho^{\prime} ; \rho^{\prime \prime}$ | $\lambda_{12}{ }^{\prime} ; \lambda_{12}{ }^{\prime \prime}$ | $\lambda_{3}{ }^{\prime} ; \lambda_{3}{ }^{\prime \prime}$ | $\eta$ | $\nabla^{2} ; \nabla^{2 \prime}$ | $G^{\prime} ; G^{\prime \prime}$ | $K^{\prime} ; K^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2.585 | 1.119 | 0.920 | 0.2579 | -1.2429 | 1.1058 | 0 | -1.3801 | 0.0472 | 0.3922 |
|  |  | 1.466 | 1.055 | 0.0861 | -0.2083 | 0.4899 | 0 | 0.0732 | 0.0566 | 0.0383 |
| 2 | 2.596 | 1.093 | 0.899 | 0.2390 | -1.1414 | 1.0105 | 0 | -1.2724 | 0.0452 | 0.3633 |
|  |  | 1.503 | 1.087 | 0.0739 | -0.1720 | 0.4292 | 0 | 0.0853 | 0.0494 | 0.0280 |
| 3 | 2.615 | 1.086 | 1.100 | 0.2482 | -1.2038 | 1.0476 | 0 | -1.3601 | 0.0434 | 0.3834 |
|  |  | 1.529 | 0.894 | 0.0704 | -0.1599 | 0.4161 | 0 | 0.0964 | 0.0473 | 0.0232 |
| 4 | 2.625 | 1.073 | 0.881 | 0.2610 | -1.2581 | 1.0649 | 0 | -1.4513 | 0.0442 | 0.4070 |
|  |  | 1.552 | 1.109 | 0.0674 | -0.1448 | 0.4026 | 0 | 0.1129 | 0.0466 | 0.0184 |
| 5 | 2.636 | 1.065 | 0.873 | 0.2589 | -1.2190 | 1.0154 | 0.000 | -1.4227 | 0.0454 | 0.4010 |
|  |  | 1.571 | 1.115 | 0.0619 | -0.1219 | 0.3686 | 0.001 | 0.1248 | 0.0442 | 0.0130 |
| 6 | 2.639 | 1.076 | 0.884 | 0.2588 | -1.2803 | 1.0823 | 0 | -1.4784 | 0.0398 | 0.4093 |
|  |  | 1.563 | 1.114 | 0.0625 | -0.1336 | 0.3816 | 0 | 0.1144 | 0.0434 | 0.0148 |
| 7 | 2.647 | 1.058 | 0.868 | 0.2684 | -1.2995 | 1.0739 | 0 | -1.5251 | 0.0433 | 0.4245 |
|  |  | 1.589 | 1.126 | 0.0598 | -0.1186 | 0.3638 | 0 | 0.1266 | 0.0428 | 0.0111 |
| 8 | 2.649 | 1.127 | 0.914 | 0.2490 | -1.0849 | 0.9130 | 0.015 | -1.2569 | 0.0493 | 0.3635 |
|  |  | 1.525 | 1.096 | 0.0844 | -0.1866 | 0.4373 | 0.004 | 0.0642 | 0.0492 | 0.0332 |
| 9 | 2.650 | 1.086 | 0.894 | 0.2873 | -1.4473 | 1.2052 | 0 | -1.6893 | 0.0380 | 0.4603 |
|  |  | 1.564 | 1.128 | 0.0623 | -0.1430 | 0.4071 | 0 | 0.1212 | 0.0456 | 0.0153 |
| 10 | 2.653 | 1.072 | 0.884 | 0.2586 | -1.2856 | 1.0839 | 0 | -1.4873 | 0.0389 | 0.4107 |
|  |  | 1.581 | 1.126 | 0.0601 | -0.1276 | 0.3689 | 0 | 0.1137 | 0.0413 | 0.0129 |
| 11 | 2.657 | 1.068 | 0.875 | 0.2655 | -1.2623 | 1.0440 | 0 | -1.4806 | 0.0435 | 0.4136 |
|  |  | 1.589 | 1.128 | 0.0623 | -0.1234 | 0.3668 | 0 | 0.1199 | 0.0430 | 0.0131 |
| 12 | 2.663 | 1.070 | 0.870 | 0.2849 | -1.3088 | 1.0176 | 0.000 | -1.5997 | 0.0413 | 0.4412 |
|  |  | 1.594 | 1.116 | 0.0613 | -0.1188 | 0.3707 | 0.001 | 0.1333 | 0.0434 | 0.0101 |
| 13 | 2.671 | 1.055 | 0.865 | 0.2767 | -1.3358 | 1.0757 | 0 | -1.5958 | 0.0403 | 0.4393 |
|  |  | 1.616 | 1.135 | 0.0553 | -0.1020 | 0.3360 | 0 | 0.1320 | 0.0403 | 0.0073 |
| 14 | 2.675 | 1.070 | 0.870 | 0.2851 | -1.3086 | 1.0167 | 0 | -1.6005 | 0.0413 | 0.4414 |
|  |  | 1.604 | 1.125 | 0.0608 | -0.1185 | 0.3686 | 0 | 0.1316 | 0.0426 | 0.0097 |
| 15 | 2.679 | 1.059 | 0.872 | 0.2671 | -1.3345 | 1.0989 | 0 | -1.5704 | 0.0370 | 0.4296 |
|  |  | 1.620 | 1.146 | 0.0536 | -0.1076 | 0.3353 | 0 | 0.1200 | 0.0374 | 0.0074 |
| 16 | 2.714 | 1.050 | 0.864 | 0.2780 | -1.4140 | 1.1359 | 0 | -1.6921 | 0.0339 | 0.4569 |
|  |  | 1.664 | 1.166 | 0.0477 | -0.0902 | 0.3025 | 0 | 0.1221 | 0.0338 | 0.0033 |
| 17 | 2.719 | 1.047 | 0.854 | 0.2852 | -1.3624 | 1.0787 | 0.000 | -1.6462 | 0.0398 | 0.4513 |
|  |  | 1.673 | 1.163 | 0.0495 | -0.0837 | 0.2855 | 0.030 | 0.1181 | 0.0341 | 0.0046 |
| 18 | 2.729 | 1.050 | 0.852 | 0.2780 | -1.2643 | 0.9906 | 0.000 | -1.5380 | 0.0434 | 0.4279 |
|  |  | 1.682 | 1.167 | 0.0491 | -0.0814 | 0.2780 | 0.033 | 0.1146 | 0.0330 | 0.0043 |
| 19 | 2.772 | 1.047 | 0.854 | 0.3246 | -1.6161 | 1.2234 | 0.010 | -2.0088 | 0.0297 | 0.5319 |
|  |  | 1.725 | 1.183 | 0.0430 | -0.0743 | 0.2723 | 0.004 | 0.1237 | 0.0308 | -0.0001 |
| 20 | 2.787 | 1.031 | 0.829 | 0.3363 | -1.5775 | 1.1216 | 0.018 | -2.0333 | 0.0307 | 0.5393 |
|  |  | 1.886 | 1.262 | 0.0328 | -0.0472 | 0.1907 | 0.001 | 0.0963 | 0.0240 | -0.0001 |
| 21 | 2.792 | 1.064 | 0.860 | 0.2934 | -1.3256 | 0.9898 | 0 | -1.6615 | 0.0384 | 0.4538 |
|  |  | 1.728 | 1.204 | 0.0506 | -0.0849 | 0.2782 | 0 | 0.1084 | 0.0320 | 0.0049 |
| 22 | 2.795 | 1.024 | 0.833 | 0.3077 | -1.4865 | 1.1276 | 0 | -1.8453 | 0.0361 | 0.4974 |
|  |  | 1.771 | 1.216 | 0.0375 | -0.0576 | 0.2304 | 0 | 0.1152 | 0.0277 | -0.0011 |
| 23 | 2.815 | 1.051 | 0.854 | 0.3225 | -1.5718 | 1.1903 | 0.010 | -1.9533 | 0.0320 | 0.5203 |
|  |  | 1.771 | 1.222 | 0.0444 | -0.0789 | 0.2635 | 0.018 | 0.1057 | 0.0284 | 0.0020 |
| 24 | 2.815 | 1.027 | 0.836 | 0.3051 | -1.4822 | 1.1389 | 0.000 | -1.8255 | 0.0357 | 0.4921 |
|  |  | 1.794 | 1.244 | 0.0397 | -0.0652 | 0.2343 | 0.029 | 0.1039 | 0.0267 | 0.0007 |
| 25 | 2.818 | 1.023 | 0.828 | 0.3029 | -1.4110 | 1.0612 | 0.000 | -1.7605 | 0.0399 | 0.4500 |
|  |  | 1.796 | 1.229 | 0.0352 | -0.0520 | 0.2134 | 0.000 | 0.1094 | 0.0258 | -0.0016 |
| 26 | 2.840 | 1.011 | 0.820 | 0.3146 | -1.5202 | 1.1487 | 0.000 | -1.8917 | 0.0378 | 0.5108 |
|  |  | 1.829 | 1.249 | 0.0320 | -0.0469 | 0.1980 | 0.000 | 0.1043 | 0.0238 | -0.0023 |
| 27 | 2.850 | 1.016 | 0.826 | 0.3157 | -1.5503 | 1.1670 | 0.000 | -1.9335 | 0.0325 | 0.5159 |
|  |  | 1.837 | 1.250 | 0.0308 | -0.0457 | 0.1924 | 0.001 | 0.1010 | 0.0228 | 0.0024 |
| 28 | 2.861 | 1.014 | 0.822 | 0.3117 | -1.4876 | 1.1283 | 0 | -1.8469 | 0.0399 | 0.5016 |
|  |  | 1.847 | 1.276 | 0.0358 | -0.0524 | 0.2031 | 0 | 0.0983 | 0.0246 | 0.0000 |
| 29 | 2.864 | 1.014 | 0.824 | 0.3176 | -1.5570 | 1.1676 | 0 | -1.9464 | 0.0322 | 0.5188 |
|  |  | 1.850 | 1.254 | 0.0292 | -0.0421 | 0.1822 | 0 | 0.0979 | 0.0219 | -0.0026 |
| 30 | 2.868 | 1.040 | 0.828 | 0.3098 | -1.3251 | 0.9189 | 0.023 | -1.7313 | 0.0441 | 0.4769 |
|  |  | 1.828 | 1.233 | 0.0369 | -0.0513 | 0.1965 | 0.054 | 0.0939 | 0.0244 | 0.0009 |
| 31 | 2.881 | 1.031 | 0.831 | 0.3351 | -1.5870 | 1.1351 | 0.017 | -2.0390 | 0.0299 | 0.5396 |
|  |  | 1.850 | 1.246 | 0.0314 | -0.0463 | 0.1923 | 0.008 | 0.0997 | 0.0228 | -0.0021 |
| 32 | 2.884 | 1.052 | 0.859 | 0.3244 | -1.6291 | 1.2500 | 0 | -2.0083 | 0.0279 | 0.5300 |
|  |  | 1.832 | 1.257 | 0.0375 | -0.0645 | 0.2215 | 0 | 0.0926 | 0.0232 | 0.0001 |
| 33 | 2.889 | 1.030 | 0.826 | 0.3268 | -1.4833 | 1.0355 | 0 | -1.9312 | 0.0338 | 0.5166 |
|  |  | 1.859 | 1.351 | 0.0311 | -0.0446 | 0.1876 | 0 | 0.0984 | 0.0227 | -0.0019 |
| 34 | 2.895 | 1.028 | 0.824 | 0.3286 | -1.4888 | 1.0346 | 0 | -1.9431 | 0.0336 | 0.5194 |
|  |  | 1.867 | 1.252 | 0.0299 | -0.0419 | 0.1804 | 0 | 0.0965 | 0.0221 | -0.0020 |
| 35 | 2.933 | 1.003 | 0.807 | 0.3296 | -1.5498 | 1.1299 | 0 | -1.9696 | 0.0365 | 0.5289 |
|  |  | 1.930 | 1.296 | 0.0256 | -0.0336 | 0.1515 | 0 | 0.0842 | 0.0191 | -0.0019 |

TABLE 2 (Continued)

|  | D | $d^{\prime} ; d^{\prime \prime}$ | $x^{\prime} ; x^{\prime \prime}$ | $\rho^{\prime} ; \rho^{\prime \prime}$ | $\lambda_{12}{ }^{\prime} ; \lambda_{12}{ }^{\prime \prime}$ | $\lambda_{3}{ }^{\prime} ; \lambda_{3}{ }^{\prime \prime}$ | $\eta$ | $\nabla^{2^{\prime}} ; \nabla^{2^{\prime \prime}}$ | $G^{\prime} ; G^{\prime \prime}$ | $K^{\prime} ; K^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 36 | 2.940 | 1.008 | 0.811 | 0.3263 | -1.5243 | 1.1153 | 0 | -1.9333 | 0.0380 | 0.5214 |
|  |  | 1.932 | 1.316 | 0.0300 | -0.0405 | 0.1641 | 0 | 0.0832 | 0.0207 | -0.0001 |
| 37 | 2.945 | 1.024 | 0.836 | 0.3069 | -1.5348 | 1.1731 | 0 | -1.8966 | 0.0297 | 0.5038 |
|  |  | 1.921 | 1.315 | 0.0277 | -0.0424 | 0.1625 | 0 | 0.0776 | 0.0177 | -0.0017 |
| 38 | 2.949 | 1.008 | 0.816 | 0.3254 | -1.5793 | 1.1682 | 0 | -1.9903 | 0.0319 | 0.5295 |
|  |  | 1.941 | 1.303 | 0.0235 | -0.0315 | 0.1427 | 0 | 0.0797 | 0.0176 | -0.0022 |
| 39 | 2.961 | 1.022 | 0.807 | 0.3303 | -1.3994 | 0.9217 | 0.027 | -1.8772 | 0.0445 | 0.5138 |
|  |  | 1.947 | 1.290 | 0.0268 | -0.0339 | 0.1496 | 0.020 | 0.0818 | 0.0196 | -0.0008 |
| 40 | 2.977 | 1.023 | 0.816 | 0.3246 | -1.4984 | 1.0193 | 0 | -1.9776 | 0.0335 | 0.5279 |
|  |  | 1.954 | 1.298 | 0.0246 | -0.0323 | 0.1439 | 0 | 0.0792 | 0.0181 | -0.0017 |
| 41 | 2.977 | 1.006 | 0.815 | 0.3248 | $-1.5616$ | 1.1568 | 0.000 | -1.9667 | 0.0334 | 0.5250 |
|  |  | 1.974 | 1.337 | 0.0252 | -0.0358 | 0.1439 | 0.015 | 0.0728 | 0.0171 | -0.0011 |
| 42 | 2.995 | 1.020 | 0.811 | 0.3376 | -1.4870 | 0.9917 | 0.006 | -1.9823 | 0.0354 | 0.5310 |
|  |  | 2.130 | 1.377 | 0.0202 | -0.0182 | 0.0995 | 0.552 | 0.0631 | 0.0147 | -0.0011 |
| 43 | 3.006 | 1.005 | 0.799 | 0.3274 | -1.4357 | 1.0022 | 0.002 | -1.8691 | 0.0434 | 0.5107 |
|  |  | 2.002 | 1.328 | 0.0221 | -0.0270 | 0.1247 | 0.007 | 0.0707 | 0.0163 | $-0.0013$ |
| 44 | 3.008 | 1.009 | 0.801 | 0.3246 | -1.4113 | 0.9832 | 0.002 | -1.8392 | 0.0488 | 0.5046 |
|  |  | 2.000 | 1.350 | 0.0265 | -0.0335 | 0.1383 | 0.005 | 0.0714 | 0.0180 | 0.0002 |
| 45 | 3.023 | 0.993 | 0.797 | 0.3348 | -1.5676 | 1.1436 | 0.000 | -1.9916 | 0.0392 | 0.5371 |
|  |  | 2.030 | 1.346 | 0.0193 | -0.0236 | 0.1132 | 0.000 | 0.0660 | 0.0148 | -0.0017 |
| 46 | 3.038 | 0.992 | 0.795 | 0.3361 | -1.5699 | 1.1425 | 0.000 | -1.9974 | 0.0394 | 0.5388 |
|  |  | 2.046 | 1.351 | 0.0183 | -0.0218 | 0.1068 | 0.000 | 0.0631 | 0.0142 | -0.0016 |
| 47 | 3.096 | 0.992 | 0.791 | 0.3432 | -1.5666 | 1.1072 | 0 | -2.0270 | 0.0383 | 0.5443 |
|  |  | 2.104 | 1.377 | 0.0168 | -0.0197 | 0.0954 | 0 | 0.0560 | 0.0129 | -0.0011 |
| 48 | 3.102 | 0.992 | 0.790 | 0.3439 | $-1.5675$ | 1.1056 | 0 | -2.0291 | 0.0385 | 0.5458 |
|  |  | 2.110 | 1.376 | 0.0162 | -0.0187 | 0.0922 | 0 | 0.0550 | 0.0126 | -0.0011 |
| 49 | 3.122 | 1.012 | 0.787 | 0.3439 | -1.3962 | 0.8540 | 0.032 | -1.9383 | 0.0490 | 0.5336 |
|  |  | 2.135 | 1.375 | 0.0182 | -0.0208 | 0.0975 | 0.019 | 0.0559 | 0.0135 | -0.0004 |
| 50 | 3.128 | 0.988 | 0.789 | 0.3407 | -1.5691 | 1.1294 | 0.000 | -2.0090 | 0.0407 | 0.5429 |
|  |  | 2.140 | 1.398 | 0.0149 | -0.0171 | 0.0850 | 0.000 | 0.0508 | 0.0116 | -0.0011 |
| 51 | 3.139 | 1.014 | 0.789 | 0.3414 | -1.3822 | 0.8456 | 0.032 | -1.9182 | 0.0496 | 0.5293 |
|  |  | 2.125 | 1.402 | 0.0218 | -0.0255 | 0.1058 | 0.018 | 0.0551 | 0.0147 | 0.0009 |
| 52 | 3.158 | 0.990 | 0.789 | 0.3395 | -1.5470 | 1.1078 | 0.001 | -1.9861 | 0.0418 | 0.5383 |
|  |  | 2.176 | 1.437 | 0.0161 | -0.0194 | 0.0870 | 0.129 | 0.0482 | 0.0115 | $-0.0006$ |
| 53 | 3.187 | 0.997 | 0.782 | 0.3391 | -1.4285 | 0.9460 | 0.003 | -1.9110 | 0.0472 | 0.5250 |
|  |  | 2.214 | 1.420 | 0.0135 | -0.0150 | 0.0745 | 0.006 | 0.0447 | 0.0105 | -0.0007 |
| 54 | 3.197 | 0.989 | 0.784 | 0.3475 | -1.5657 | 1.0921 | 0 | -2.0394 | 0.0395 | 0.5493 |
|  |  | 2.208 | 1.425 | 0.0131 | -0.0147 | 0.0738 | 0 | 0.0443 | 0.0103 | -0.0008 |
| 55 | 3.207 | 0.997 | 0.782 | 0.3394 | -1.4213 | 0.9339 | 0.003 | -1.9087 | 0.0478 | 0.5250 |
|  |  | 2.210 | 1.418 | 0.0135 | -0.0149 | 0.0747 | 0.007 | 0.0449 | 0.0105 | -0.0007 |
| 56 | 3.208 | 0.999 | 0.802 | 0.3363 | -1.5917 | 1.1525 | 0 | -2.0309 | 0.0330 | 0.5407 |
|  |  | 2.210 | 1.455 | 0.0141 | -0.0171 | 0.0772 | 0 | 0.0430 | 0.0099 | -0.0009 |
| 57 | 3.317 | 0.995 | 0.776 | 0.3424 | -1.4141 | 0.9121 | 0.004 | -1.9161 | 0.0494 | 0.5284 |
|  |  | 2.325 | 1.474 | 0.0106 | -0.0114 | 0.0588 | 0.005 | 0.0360 | 0.0083 | -0.0007 |
| 58 | 3.323 | 1.007 | 0.774 | 0.3497 | -1.3745 | 0.7962 | 0.036 | -1.9529 | 0.0527 | 0.5409 |
|  |  | 2.324 | 1.462 | 0.0117 | -0.0126 | 0.0634 | 0.018 | 0.0382 | 0.0089 | -0.0006 |
| 59 | [3.330 | 1.016 | 0.802 | 0.3437 | -1.4903 | 0.9649 | 0 | -2.0156 | 0.0348 | 0.5387 |
|  |  | 2.284 | 1.481 | 0.0131 | -0.0153 | 0.0693 | 0 | 0.0387 | 0.0090 | -0.0006] |
| 60 | 3.332 | 1.007 | 0.773 | 0.3499 | -1.3703 | 0.7889 | 0.037 | -1.9519 | 0.0531 | 0.5410 |
|  |  | 2.340 | 1.464 | 0.0111 | -0.0117 | 0.0608 | 0.017 | 0.0372 | 0.0086 | $-0.0007$ |
| 61 | 3.459 | 1.006 | 1.769 | 0.3512 | -1.3587 | 0.7673 | 0.039 | -1.9502 | 0.0545 | 0.5421 |
|  |  | 2.456 | 1.524 | 0.0086 | -0.0089 | 0.0475 | 0.017 | 0.0297 | 0.0066 | -0.0008 |
| 62 | 3.478 | 0.988 | 0.778 | 0.3451 | -1.4952 | 1.0192 | 0.002 | -1.9712 | 0.0450 | 0.5378 |
|  |  | 2.491 | 1.586 | 0.0075 | -0.0082 | 0.0427 | 0.003 | 0.0263 | 0.0058 | -0.0008 |
| 63 | 3.494 | 0.985 | 0.777 | 0.3520 | -1.5568 | 1.0663 | 0 | -2.0474 | 0.0413 | 0.5531 |
|  |  | 2.509 | 1.594 | 0.0073 | -0.0079 | 0.0413 | 0 | 0.0255 | 0.0056 | $-0.0008$ |
| 64 | [3.642 | 0.993 | 0.769 | 0.3453 | -1.3946 | 0.8716 | 0.004 | -1.9175 | 0.0519 | 0.5313 |
|  |  | 2.651 | 1.658 | 0.0054 | -0.0056 | 0.0319 | 0.008 | - 0.0205 | 0.0042 | -0.0008] |
| 65 | [3.793 | 1.005 | 0.763 | 0.3521 | -1.3376 | 0.7343 | 0.041 | -1.9407 | 0.0567 | 0.5419 |
|  |  | 2.794 | 1.716 | 0.0043 | -0.0042 | 0.0250 | 0.018 | 0.0165 | 0.0033 | -0.0008] |
| 66 |  | 1.012 | 0.793 | 0.3490 | -1.4856 | 0.9309 | 0 | -2.0404 | 0.0357 | 0.5458 |
| 67 |  | 1.025 | 0.827 | 0.3558 | -1.7527 | 1.2500 | 0 | -2.2534 | 0.0198 | 0.5836 |

${ }^{a}$ Numbering as in Table 1. Upper line, $\mathrm{N}_{\mathrm{d}}-\mathrm{H}$ bond (single prime); lower line, $\mathrm{H} \cdots \mathrm{N}_{\mathrm{a}}$ bond (double prime). $\rho^{\prime}=\rho_{\mathrm{c}}\left(\mathrm{X}^{\prime}\right), \rho^{\prime \prime}=\rho_{\mathrm{c}}\left(\mathrm{X}^{\prime \prime}\right), \nabla^{2}=$ $\nabla^{2} \rho_{\mathrm{c}}\left(\mathrm{X}^{\prime}\right), \lambda_{12}{ }^{\prime}=\left(\lambda_{1}{ }^{\prime}+\lambda_{2}{ }^{\prime}\right) / 2$ etc. Ellipticity $\eta=\left(\lambda_{1} / \lambda_{2}\right)-1$.
$=D / 2$, which corresponds to the $45^{\circ}$ symmetry line in Figure 1A. The scatter about this line reflects the departure of the $\mathrm{N}_{\mathrm{d}} \mathrm{HN}_{\mathrm{a}}$ angles from $180^{\circ}$ : the mean deviation from the $D / 2$ line is $0.013 \AA\left(r^{2}=0.989\right)$ but only $0.003 \AA$ when the two outliers 20 and 42 with $\mathrm{N}_{\mathrm{d}} \mathrm{HN}_{\mathrm{a}}<145^{\circ}$ are removed. The $d^{\prime}$ branch of this plot converges with increasing $D$ slowly to the limiting value $d_{0}$. The $d^{\prime \prime}$ distance increases with $D$ at first nonlinearly and
then to all intent and purpose linearly. The variation $\partial d^{\prime \prime} / \partial D=$ $(1-E) /(1-2 E), E=\exp \left[\left(d_{0}-d^{\prime \prime}\right) / b\right]$, which tends to infinity as $d^{\prime \prime} \rightarrow d_{\text {sym }}$ and to unity as $d^{\prime \prime} \rightarrow \infty$, amounts to 1.09 at $D=$ $3 \AA$ (corresponding to $d^{\prime \prime} \sim 2 \AA$ ) and to 1.005 at $D=4 \AA$ (corresponding to $d^{\prime \prime} \sim 3 \AA$ ).

The distance $d^{\prime}=1.02 \AA$, which, in structural comparisons and in the absence of accurate experimental determination, is


Figure 1. (A) HF data set: regression of $d^{\prime}$ on $d^{\prime \prime}$ according to (1) + (4) and of $x^{\prime}$ on $x^{\prime \prime}$ according to (6). (B) MP2 data set: regression of $d^{\prime}$ on $d^{\prime \prime}$ according to $(1)+(4)$ and of $x^{\prime}$ on $x^{\prime \prime}$ according to (6a). The regression functions are symmetric about the $45^{\circ}$ line.


Figure 2. Correlation of $d$ with $D$ for the HF (A) and MP2 (C) data sets and of $x$ with $D$ for the HF (B) and MP2 (D) data sets. The straight lines are the respective $\left(d^{\prime}+d^{\prime \prime}\right) / 2$ and $\left(x^{\prime}+x^{\prime \prime}\right) / 2$ loci. The two outliers $a$ and $b$ in A represent the $d^{\prime \prime}$ of $\mathbf{2 0}$ and 42, respectively, in which NHN $<145^{\circ}$.
often assumed for the length of the $\mathrm{N}_{\mathrm{d}}-\mathrm{H}$ bond, is reached at $D=2.91 \AA$; for the Steiner set, the corresponding value would be $3.08 \AA$.

A parallel examination of the $d^{\prime}, d^{\prime \prime}$ relationship was undertaken for the MP2-optimized species (Tables 3 and 4, Figures 1B and 2C, and text below). This resulted in

$$
\begin{align*}
d_{0}=1.009(8) \AA, b= & 0.365(13) \AA, d_{\mathrm{sym}}= \\
& 1.262(17) \AA, D_{\min }=2.524(34) \AA \tag{4a}
\end{align*}
$$

( $r^{2}=0.978, \sigma=0.015 \AA \sim 1.4 \%$ of range, $r\left(d_{0}, b\right)=-0.95$; Figure 1B). The mean deviation from the $D / 2$ line in Figure 2C was $0.007 \AA, r^{2}=0.999$. Testing the sums of squares of the residuals showed that, unlike for the HF data set and (1) + (4), representing the MP2 set by (1) $+(2)$ is statistically equivalent to the regression (1) $+(4 a)$. The difference between Steiner's model function and (1) $+(4 a)$ is everywhere negative. It varies from $-0.012 \AA$ at $d^{\prime \prime}=2.5 \AA$ to $-0.004 \AA$ at $d_{\text {sym }}$ with a minimum at $d^{\prime \prime}=1.31 \AA$ and a residual value of ca. $-0.013 \AA$ for $d^{\prime \prime}>3 \AA$. The MP2 optimization thus overvalues the $d^{\prime}$, $d^{\prime \prime}$, but the MP2-optimized distances are closer to experiment than the corresponding HF values.

The HF optimizations of the very weakly H-bonded 64 and 65 converged, but they resulted in negative frequencies and very large $D$ and $d^{\prime \prime}$ values. Here the MP2 optimization was successful: significantly, the $D$ and $d^{\prime \prime}$ values it yielded were $0.27-0.37 \AA$ smaller than those from the HF optimizations.

When 59, 64, and 65 (for which proper HF optimization could not be achieved) are excluded from the HF and MP2 sets, the difference between $D(\mathrm{HF})$ and $D(\mathrm{MP} 2)$, statistically, is practically constant and equal to $\sim 0.12 \AA$.

While the above $d^{\prime}, d^{\prime \prime}$ correlations pertain specifically to $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds, without consideration of acceptor atoms other than nitrogen, they can be shown to be consistent with the more comprehensive scheme elaborated for $\mathrm{N}-\mathrm{H}-\mathrm{X}$ bonds; ${ }^{16}$ i.e., their validity is not merely local (for a discussion of local vs "universal" relations involving properties at bond-critical points, see ref 17). When the $d$ vs $D$ plot of Figures 2A,C is redrawn as a $d^{\prime \prime}-d^{\prime}$ vs $D$ plot, the resulting curve represents a special case in the family of the $r_{1}+r_{2}(\rightarrow D)$ vs $r_{1}-r_{2}\left(\rightarrow d^{\prime \prime}-d^{\prime}\right)$ curves for $\mathrm{N}-\mathrm{H}-\mathrm{X}$ bonds, where $\mathrm{X}=\mathrm{F}, \mathrm{Cl}, \mathrm{Br}, \mathrm{CN}$ and $r_{1}=$ $d(\mathrm{H} \cdots \mathrm{X}), r_{2}=d(\mathrm{~N}-\mathrm{H}) .{ }^{16}$ With ab initio (RHF/6-31G** or better) $r_{1}, r_{2}$ values, the curve for each of the $\mathrm{N}, \mathrm{X}$ combinations is very well fitted by the generalized form of (3),

$$
\begin{array}{r}
r_{1}+r_{2}=2 r_{02}+\left(r_{1}-r_{2}\right)+2 b \ln \left\{1+\exp \left[\left(r_{01}-r_{02}-\right.\right.\right. \\
\left.\left.\left.r_{1}+r_{2}\right) / b\right]\right\}  \tag{3a}\\
b=\left[\left(r_{1}+r_{2}\right)_{\min }-\left(r_{01}+r_{02}\right)\right] /(2 \ln 2)
\end{array}
$$

$r_{01}, r_{02}\left(\rightarrow d_{0}\right)$ being the limiting values of $r_{1}, r_{2}$. Our examination of the fitted curves shows that their minima are on a locus approximated by the straight line $\left(r_{1}+r_{2}\right)_{\min }=2.538(14)+$ $1.022(57)\left(r_{1}-r_{2}\right), n=9, r^{2}=0.979, \sigma=0.036 \AA \sim 6 \%$ of range. The $\left(r_{1}+r_{2}\right)_{\min }$ value estimated for the special case of homoconjugated bonds, i.e., $\mathrm{X}=\mathrm{N}$ and $r_{1}-r_{2}=0$, would thus be 2.538 (14) $\AA$, in excellent agreement with the value estimated for $D_{\min }$ from (4a) for the MP2 set with Steiner's $D_{\text {min }}$ from (2).

The Symmetric $\mathbf{N}-\mathbf{H}-\mathbf{N}$ Bond. Steiner's experimental regression $(1)+(2)$ and Figures 1 and 2 imply that, within the natural variability of the sample set, the conjugate $d^{\prime}, d^{\prime \prime}$ distances in linear or near-linear $\mathrm{Z}_{\mathrm{d}} \mathrm{N}_{\mathrm{d}}-\mathrm{H} \cdots \mathrm{N}_{\mathrm{a}} \mathrm{Z}_{\mathrm{a}}$ bonds are uniquely determined by $D$ and that all such $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds are asymmetric except at $D_{\min }$ where $d^{\prime}=d^{\prime \prime}=d_{\text {sym }}$. A further implication is that any linear symmetric $\mathrm{ZN}-\mathrm{H}-\mathrm{NZ}$ bond will, within the regression uncertainty, have the same dimensions, regardless of the nature of Z , i.e., of the particular chemistry of the molecule or ion containing the bond. The centrosymmetric bond in [4.4.4] ${ }^{+}$, with its $D \sim 2.54 \AA$ (see above), is a documented example of such a bond. The available evidence ${ }^{5-7,9}$ indicates that this, the shortest linear $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bond known, has a single potential minimum.

Given the close agreement between Steiner's experimental regression and that of (1) $+(4)$, convergence to a linear symmetric bond would be expected for the potentially symmetric (= prosymmetric) species of $\mathbf{S}$ on HF optimization. However, as evident from Figures 1A and 2A, this expectation did not realize.

In Table 1, entries $\mathbf{2 - 5}, \mathbf{7}, \mathbf{1 1}, \mathbf{2 1}, \mathbf{2 3}, \mathbf{3 0}$, and $\mathbf{3 2}$ are prosymmetric. As the table shows, they all optimized properly but not in point-group symmetries that require symmetric, though not necessarily linear, $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds. None of these species has a $D$ value below $2.5 \AA$ (in fact, the smallest $D$ of any of the species in $\mathbf{S}$ is $2.585 \AA$, in $\mathbf{1}$ ), and their $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds are distinctly asymmetric. The position of the H atom in

TABLE 3: MP2/6-31G**-Optimized Equilibrium Molecular Geometries of Prosymmetric and Other Selected Species of Table $1^{a}$

| configuration | PG | $\mathrm{N}_{\mathrm{d}} \mathrm{HN}_{\mathrm{a}}$ | -E | - $E_{\text {f }}$ | $\epsilon\left(\mathrm{N}_{\mathrm{d}}\right)$ | $\epsilon(\mathrm{H})$ | $\epsilon\left(\mathrm{N}_{\mathrm{a}}\right)$ | other PGs tried |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 [FCNH $\cdots$ NCF] ${ }^{+}$ | $D_{\infty}$ | (180) | 384.62456 | 0.04846 | -0.473 | 0.621 | -0.473 | $C_{\text {ovv }}, C_{\text {s }}$ |
| 7 [SCNH $\cdots$ NCS] ${ }^{-}$ | $C_{\infty}$ | (180) | 981.13549 | 0.04834 | -0.459 | 0.449 | -0.474 | [ $D_{\text {oh }}$ ], $C_{1}$ |
| $4[\mathrm{LiCNH} \cdots \mathrm{NCLi}]^{+}$ | $D_{\infty}$ | (180) | 200.55692 | 0.06488 | -0.392 | 0.483 | -0.392 | $C_{\infty}{ }^{\circ}, C_{\text {s }}$ |
| 3 [ $\mathrm{HCNH} \cdots \mathrm{NCH}]^{+}$ | $D_{\infty}$ | (180) | 186.66584 | 0.05110 | -0.349 | 0.527 | -0.349 | $C_{\infty \nu}, C_{2 v}, C_{s},\left[C_{2}\right]$ |
| $10\left[\mathrm{NCCNH} \cdots \mathrm{NCCN}^{+}\right.$ | $D_{\text {© }}$ | (180) | 370.61388 | 0.03871 | -0.443 | 0.524 | -0.443 | $C_{\infty \nu}, C_{1}$ |
| 11 [CNH $\cdots$ NC] ${ }^{-}$ | $C_{s}$ | 180.0 | 185.75750 | 0.06025 | -0.615 | 0.532 | -0.638 | [ $D_{\infty}$ ] $],\left[C_{\infty \sim v}\right],\left[C_{2 v}\right]$ |
| 17 [CNH $\cdots \cdot \mathrm{NNN}]^{-}$ | $C_{\text {s }}$ | 177.2 | 256.97476 | 0.05069 | -0.374 | 0.396 | -0.535 | [ $C_{\infty}$ ] $]$ |
| $1\left[\mathrm{~F}_{3} \mathrm{NH} \cdots \mathrm{NCF}\right]^{+}$ | $C_{3 v}$ | (180) | 545.66829 | 0.05940 | -0.304 | 0.512 | -0.577 |  |
| $23\left[\mathrm{~F}_{2} \mathrm{HNH} \cdots \mathrm{NHF}_{2}\right]^{+}$ | $C_{s}$ | 176.9 | 508.83071 | 0.03307 | 0.243 | 0.498 | 0.130 |  |
| $20^{b}\left[\mathrm{FH}_{2} \mathrm{NH} \cdots \mathrm{NH}_{2} \mathrm{~F}\right]^{+}$ | $C_{s}$ | 157.4 | 310.98011 | 0.04388 | -0.184 | 0.446 | -0.336 |  |
| $32\left[\mathrm{~F}_{3} \mathrm{NH} \cdot \cdots \mathrm{NF}_{3}\right]^{+}$ | $C_{3 v}$ | (180) | 706.70503 | 0.02006 | 0.676 | 0.500 | 0.577 | [ $D_{3 d}$ ], [ $C_{1}$ ] |
| $21\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NH}_{3}\right]^{+}$ | $C_{3 v}$ | (180) | 113.16684 | 0.04995 | -0.708 | 0.445 | -0.814 | [ $D_{3 d}$ ], $C_{1}$ |
| 53 OCNH $\cdots$ NCH | $C_{s}$ | 179.7 | 261.40659 | 0.00969 | -0.575 | 0.354 | -0.319 | [ $C_{\infty}$ ] |
| $59\left[\mathrm{H}_{3} \mathrm{NH} \cdots \mathrm{NF}_{3}\right]^{+}$ | $C_{3 v}$ | (180) | 409.96801 | 0.00756 | -0.652 | 0.402 | 0.609 | $C_{1}$ |
| $64 \mathrm{OCNH} \cdots \mathrm{NF}_{3}$ | $C_{s}$ | 176.5 | 521.46024 | 0.00275 | -0.568 | 0.324 | 0.658 | $\left.{ }_{\left[C_{3 v}\right.}\right]$ |
| $65^{\circ} \mathrm{NNNH} \cdots{ }^{\text {N }} \mathrm{NF}_{3}$, anti | $C_{s}$ | 173.3 | 517.58028 | 0.00251 | -0.418 | 0.317 | 0.658 | ${ }_{[C 1]}$ |
| $65^{\text {c }}$ NNNHH $\cdots \mathrm{NF}_{3}$, syn | $C_{s}$ | 178.2 | 517.58028 | 0.00251 | -0.418 | 0.317 | 0.658 | $\left[C_{3 v}\right]$ |
| $66 \mathrm{H}_{3} \mathrm{NH}^{+}$ | $T_{d}$ |  | 56.73368 |  | -0.631 | 0.408 |  |  |
| $67 \mathrm{~F}_{3} \mathrm{NH}^{+}$ | $C_{3 v}$ |  | 353.45819 |  | 0.783 | 0.454 |  |  |

${ }^{a}$ No negative frequencies. Arranged in the order of increasing $D$. Numbering, symbols, and abbreviations as in Table 1. ${ }^{b}$ Planar ring with $\mathrm{F} \cdots 2 \mathrm{H}$ bifurcation. ${ }^{c}$ Syn with respect to the F atom in the symmetry plane and closing the $\mathrm{NNNH} \cdots \mathrm{N}-\mathrm{F}$ arc.

TABLE 4: Internuclear and Atom $\cdots$ BCP Distances ( $(\AA)$, Electron Densities $\rho_{\mathrm{c}}(r)$, Laplacians $\nabla^{2} \rho_{\mathrm{c}}(r)$, Eigenvalues $\lambda_{\mathrm{i}}$ of the Laplacians, Ellipticities $\eta$, and Kinetic Energy Densities $G(r)$ and $K(r)$ at the Bond-Critical Points (all in au) in the MP2/ 6-31G** Optimized Species of Table 3 (Arranged in the Order of Increasing D) ${ }^{a}$

|  | D | $d^{\prime} ; d^{\prime \prime}$ | $x^{\prime} ; x^{\prime \prime}$ | $\rho^{\prime} ; \rho^{\prime \prime}$ | $\lambda_{12}{ }^{\prime} ; \lambda_{12}{ }^{\prime \prime}$ | $\lambda_{3}{ }^{\prime} ; \lambda_{3}{ }^{\prime \prime}$ | $\eta$ | $\nabla^{2} ; \nabla^{2^{\prime \prime}}$ | $G^{\prime} ; G^{\prime \prime}$ | $K^{\prime} ; K^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 2.509 | 1.255 | 0.983 | 0.1478 | -0.5023 | 0.5657 | 0 | -0.4388 | 0.064 | 0.1736 |
|  |  | 1.255 | 0.983 | 0.1478 | -0.5023 | 0.5657 | 0 | -0.4388 | 0.064 | 0.1736 |
| 7 | 2.510 | 1.204 | 0.934 | 0.1782 | -0.6031 | 0.5964 | 0 | -0.6098 | 0.070 | 0.2222 |
|  |  | 1.307 | 0.975 | 0.1346 | -0.3849 | 0.5481 | 0 | -0.2218 | 0.069 | 0.1246 |
| 4 | 2.518 | 1.259 | 0.959 | 0.1566 | -0.4948 | 0.5682 | 0 | -0.4213 | 0.069 | 0.1741 |
|  |  | 1.259 | 0.959 | 0.1566 | -0.4948 | 0.5682 | 0 | -0.4213 | 0.069 | 0.1741 |
| 3 | 2.520 | 1.260 | 0.964 | 0.1538 | -0.4950 | 0.5647 | 0 | -0.4253 | 0.066 | 0.1727 |
|  |  | 1.260 | 0.964 | 0.1538 | -0.4950 | 0.5647 | 0 | -0.4253 | 0.066 | 0.1727 |
| 10 | 2.524 | 1.262 | 0.964 | 0.1522 | -0.4877 | 0.5588 | 0 | -0.4165 | 0.066 | 0.1697 |
|  |  | 1.262 | 0.964 | 0.1522 | -0.4877 | 0.5588 | 0 | -0.4165 | 0.066 | 0.1697 |
| 11 | 2.534 | 1.267 | 0.986 | 0.1491 | -0.4828 | 0.5428 | 0.000 | -0.4228 | 0.065 | 0.1703 |
|  |  | 1.267 | 0.986 | 0.1491 | -0.4828 | 0.5428 | 0.000 | -0.4228 | 0.065 | 0.1703 |
| 17 | 2.617 | 1.097 | 0.874 | 0.2487 | -1.0144 | 0.8323 | 0.001 | -1.1966 | 0.058 | 0.3571 |
|  |  | 1.521 | 1.065 | 0.0763 | -0.1569 | 0.4038 | 0.030 | 0.0900 | 0.048 | 0.0257 |
| 1 | 2.629 | 1.111 | 0.886 | 0.2326 | -0.9533 | 0.8227 | 0 | $-1.0840$ | 0.059 | 0.3294 |
|  |  | 1.518 | 1.091 | 0.0848 | -0.2066 | 0.4522 | 0 | 0.0390 | 0.047 | 0.0377 |
| 23 | 2.637 | 1.139 | 0.902 | 0.2480 | -0.9996 | 0.8154 | 0.008 | -1.1839 | 0.059 | 0.3547 |
|  |  | 1.499 | 1.069 | 0.0937 | -0.2254 | 0.4932 | 0.010 | 0.0425 | 0.053 | 0.0423 |
| 20 | 2.653 | 1.086 | 0.865 | 0.2822 | -1.1941 | 0.9070 | 0.014 | -1.4811 | 0.051 | 0.4211 |
|  |  | 1.618 | 1.122 | 0.0661 | -0.1291 | 0.3723 | 0.009 | 0.1142 | 0.043 | 0.0139 |
| 32 | 2.659 | 1.144 | 0.908 | 0.2455 | -1.0044 | 0.8349 | 0 | $-1.1738$ | 0.057 | 0.3503 |
|  |  | 1.516 | 1.079 | 0.0894 | -0.2162 | 0.4760 | 0 | 0.0436 | 0.049 | 0.0383 |
| 21 | 2.669 | 1.135 | 0.897 | 0.2390 | -0.9161 | 0.7137 | 0 | -1.1186 | 0.057 | 0.3366 |
|  |  | 1.534 | 1.087 | 0.0842 | -0.1790 | 0.4242 | 0 | 0.0662 | 0.048 | 0.0314 |
| 53 | 3.087 | 1.011 | 0.785 | 0.3240 | -1.3009 | 0.8614 | 0.008 | $-1.7403$ | 0.057 | 0.4925 |
|  |  | 2.076 | 1.343 | 0.0196 | -0.0333 | 0.1048 | 0.006 | 0.0582 | 0.014 | -0.0004 |
| 59 | 3.123 | 1.032 | 0.810 | 0.3233 | -1.3424 | 0.8858 | 0 | -1.7990 | 0.045 | 0.4947 |
|  |  | 2.090 | 1.368 | 0.0209 | -0.0267 | 0.1085 | 0 | 0.0551 | 0.013 | -0.0004 |
| 64 | 3.373 | 1.006 | 0.772 | 0.3308 | -1.2871 | 0.8135 | 0.009 | -1.7607 | 0.062 | 0.5018 |
|  |  | 2.369 | 1.495 | 0.0109 | -0.0121 | 0.0581 | 0.006 | 0.0339 | 0.008 | -0.0007 |
| 65 | 3.430 | 1.020 | 0.773 | 0.3273 | -1.2212 | 0.7267 | 0.037 | -1.7157 | 0.065 | 0.4942 |
|  |  | 2.415 | 1.509 | 0.0103 | -0.0112 | 0.0546 | 0.014 | 0.0319 | 0.007 | -0.0007 |
| 65 | 3.440 | 1.020 | 0.773 | 0.3272 | -1.2203 | 0.7259 | 0.037 | -1.7148 | 0.065 | 0.4940 |
|  |  | 2.421 | 1.512 | 0.0102 | -0.0111 | 0.0537 | 0.013 | 0.0315 | 0.007 | -0.0007 |
| 66 |  | 1.023 | 0.795 | 0.3331 | -1.3629 | 0.8594 |  | -1.8665 | 0.045 | 0.5119 |
| 67 |  | 1.042 | 0.831 | 0.3317 | -1.5195 | 1.0831 |  | -1.9559 | 0.031 | 0.5198 |

${ }^{a}$ For numbering, symbols, etc. see Tables 2 and 3 .
the bond is plotted against $D$ in Figure 3. Interestingly, the resulting $d^{\prime}$ and $d^{\prime \prime}$ in these bonds are well accommodated by the $d$ vs $D$ curve of Figure 2A, and the separation $z=2 d^{\prime \prime}-$ $D$ of the two alternative positions of the H atom follows directly from (3) $+(4)$.

When attempts were made to optimize these species in centrosymmetric point groups or in point groups containing $C_{s^{-}}$ $(\perp \mathrm{NN})$ or $C_{2}(\perp \mathrm{NN})$ symmetry elements (cf. Table 1$)$ and from different parameter estimates, the optimizations converged but resulted in one or more negative frequencies and higher total


Figure 3. HF (left) and MP2 (right) optimizations of the prosymmetric species: position of the H atom in the linear or near-linear $\mathrm{ZN}-\mathrm{H}-$ NZ bond plotted against $D$. Large circles: N atoms. Small circles: H atoms. Only one of the alternative positions (relative to the midpoint of the $\mathrm{N} \cdots \mathrm{N}$ distance) is shown; the two positions are separated by a distance $z=d^{\prime \prime}-(D / 2)$ (see text). The dashed lines relate the HFoptimized to the MP2-optimized geometries. The shaded bands represent the $D_{\min } \pm 1 \sigma$ limits as estimated from (4) and (4a), respectively, and the $\pm 1 \sigma$ estimated from the experimental $D$ values for the shortest known, symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bond (XD, X-ray diffraction at room temperature; ND , neutron diffraction at 20 K ; see text).
electronic energies. However, the $D$ values so obtained were significantly closer to the $D_{\text {min }}$ estimated from (1) + (4) than the values in the corresponding, properly optimized but asymmetric conformations.

These observations led us to believe that the failure to obtain symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds in the $\mathbf{S}$ set was an artifact of the HF method and that symmetric bonds might be found in at least some of the above species if their geometries were optimized at a higher level. The prosymmetric species were therefore reoptimized in MP2/6-31G** in several point-group symmetries each, with the lowest energy conformation reported in Table 3.

The pro- $D_{\infty h}$ species 2-4 and $\mathbf{1 0}$ (but not $\mathbf{5}$ and $\mathbf{3 0}$ ) optimized properly under that symmetry. Numbers $\mathbf{7}$ and $\mathbf{1 1}$ each gave one negative frequency in $D_{\infty h}$, but $\mathbf{1 1}$ optimized in $\mathrm{C}_{\mathrm{s}}$ rendered a geometry indistinguishable from $D_{\infty h}$, while 7, when optimized in $C_{\infty \nu}$, remained slightly asymmetric. However, MP2 optimization of $\mathbf{5}$ and $\mathbf{3 0}$, while convergent, resulted in imaginary frequencies for all the geometries and point groups tried (note that the HF optimizations in $C_{s}$ were successful (cf. Table 1).

The less strongly H-bonded ( $D>2.65 \AA$ ) pro- $D_{3 d}$ species 21 and 32, however, resisted proper MP2 optimization in that symmetry, and their $C_{3 v}$-optimized (staggered) geometries contained unmistakably asymmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds. Similarly, optimization of the pro- $C_{2 h} 23$ in $C_{s}$ (pseudo- $C_{2 h}$ ) resulted in an asymmetric $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bond both in HF and MP2. Since the equilibrium geometries of weakly H-bonded complexes appear to change only insignificantly in post-MP2 optimizations, ${ }^{18}$ this asymmetry confirms the validity of the HF results, viz. the (static) $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds in 21, 23, and $\mathbf{3 2}$ are asymmetric. This agrees with the expectation from Figure 2A and, for 21, also with previous ${ }^{10}$ ab initio calculations that included varying amounts of correlation effects (lowest $E=-112.87099$ au, smallest $D=2.73 \AA, d^{\prime}=1.144 \AA$; to be compared with $-113.166855 \mathrm{au}, 2.669 \AA, 1.135 \AA$ in $\left.21\left(\mathrm{MP} 2 / C_{3 v}\right)\right) .{ }^{19}$ For 21, there are also experimental $D$ values for the $\mathrm{N}_{2} \mathrm{H}_{7}{ }^{+}$cation from the careful determinations of the crystal structures of
several phases of $\mathrm{N}_{2} \mathrm{H}_{7} \mathrm{I}$ and $\mathrm{N}_{2} \mathrm{D}_{7} \mathrm{I}$ by X-ray (powder and single crystal) and neutron powder diffraction at appropriate temperatures $(\geq 160 \mathrm{~K}) .{ }^{21}$ Comparisons with 21 are complicated by the presence of orientational disorder in some of the phases and by the bond lengthening which is caused by the nonnegligible $\mathrm{N}-\mathrm{H}^{+} \cdots \mathrm{I}^{-}$interactions, but in the phase most suitable for a comparison, the tetragonal $\mathrm{N}_{2} \mathrm{H}_{7} \mathrm{I}(I I)$ at $207 \mathrm{~K},{ }^{21 a}$ the $\mathrm{N} \cdots \mathrm{N}$ distance of 2.68(6) $\AA$ in the asymmetric cation compares well with the $D$ (MP2) in 21, $2.669 \AA$.

The relationship between the HF and the MP2 optimizations of the prosymmetric species is displayed in Figure 3. The $D$ (MP2) values are all substantially smaller than the corresponding $D(\mathrm{HF})$ values, those for $\mathbf{2 - 4}, \mathbf{7}, \mathbf{1 0}$, and $\mathbf{1 1}$ by $0.09-$ $0.13 \AA$, those for 21, 23, and 32 by $0.13-0.22 \AA$; i.e., the difference appears to increase roughly linearly with $D(\mathrm{HF})$. The $D(\mathrm{HF})$ values for the first group on MP2 reoptimization dropped into a band $\sim 0.025 \AA$ wide, with a $D($ MP2 $)$ mean of $2.519(9)$ $\AA$. This band is well within the $\pm 1 \sigma$ range, $2.50-2.55 \AA$, estimated for the $D_{\min }$ (MP2) limit from the $d^{\prime}, d^{\prime \prime}$ correlation (4a). It is also close to the $D$ found for the linear symmetric bond in $[4.4 .4]^{+}$, especially if allowance were to be made for the effect temperature may have on the steric constraint imposed on the bond by the compact cation framework and the crystal environment.

As noted above, the four cations 2-4 and $\mathbf{1 0}$ optimized in MP2/ $D_{\infty h}$ symmetry without difficulty. For the anions 7 and 11, proper MP2 optimization was not achieved in $D_{\infty h}$ but optimization in lower symmetry resulted in an effectively $D_{\infty h}$ symmetry for 11 and nearly $D_{\infty h}$ for 7. All of these species contain $-\mathrm{C} \equiv \mathrm{N}-\mathrm{H}-\mathrm{N} \equiv \mathrm{C}-$ as the central group. However, for the anions $[\mathrm{NNN}-\mathrm{H}-\mathrm{NNN}]^{-}(\mathbf{3 0})$ and $\left[\mathrm{OCN}-\mathrm{H}-\mathrm{NCO}^{-}(\mathbf{5})\right.$ the MP2 optimization failed in all the symmetries tried (Table 3 ), even though the geometries could be optimized as quasilinear asymmetric in $\mathrm{HF} / C_{s}$. The above results, although limited, seem to suggest that the tendency of prosymmetric species to form symmetric $\mathrm{ZN}-\mathrm{H}-\mathrm{NZ}$ conformations can be related to the type of orbital hybridization on the N atom as determined by the order of the bond(s) between the N atom and the atom covalently attached to it; ionic charge undoubtedly also plays a role. This tendency appears to increase with the order of that bond in the $\mathrm{ZN}-\mathrm{H}$ donor species. Thus, for $\mathrm{ZC} \equiv \mathrm{N}-\mathrm{H}$, the triple bond gives rise to a symmetric bond, while $\left[\mathrm{Z}_{3} \mathrm{~N}-\mathrm{H}\right]^{+}$, at the other end, generates an asymmetric $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bond in the deprotonated dimer. ${ }^{22}$ This dependence of the strength, as measured by the internuclear distance, of the unengaged $\mathrm{N}-\mathrm{H}$ bond in the donor species on bond order parallels that of the $\mathrm{C}-\mathrm{H}$ bond in gasphase $\mathrm{CH}_{4}, \mathrm{C}_{2} \mathrm{H}_{4}, \mathrm{C}_{2} \mathrm{H}_{2}$, and HCN molecules, in which the experimental $\mathrm{C}-\mathrm{H}$ bond length ${ }^{23}$ correlates very well with the bond order $s: d(\mathrm{C}-\mathrm{H})=1.0934 s^{-0.02907}, \sigma=0.0007 \AA \sim 2 \%$ of the $d(\mathrm{C}-\mathrm{H})$ range.

The Symmetric $\mathbf{N}-\mathbf{H}-\mathbf{N}$ Bond: Ab Initio vs Experiment. The long-standing curiosity about the symmetry of short, homoconjugated $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds is reflected in the variety of experimental as well as theoretical investigations, ${ }^{10,20,21,24}$ even though these are not nearly as numerous as for the corresponding $\mathrm{O}-\mathrm{H}-\mathrm{O}$ bonds. For comparison with the above conclusions from the MP2 optimizations we present results of pertinent crystallographic studies for $D<2.9 \AA$ in Figure 4 and Table 5, where $z / 2$ is the displacement of the H atom (or the proton) from the center of the $\mathrm{N} \cdots \mathrm{N}$ distance. This selection is not as homogeneous as one might wish, since it contains determinations by neutron as well as X-ray diffraction at a variety of temperatures, and it includes, of necessity, bonds with NHN angles stated to be considerably smaller than $180^{\circ}$ (Table 5).

TABLE 5: Experimental Data for Homoconjugated (Prosymmetric) ZN-H-NZ Bonds Included in Figure $4^{a}$

|  | refcode ${ }^{b}$ | method | $D, \AA$ | $d^{\prime}, \AA$ | $d^{\prime \prime}, \AA$ | NHN | $z / 2^{c}$ | $(z / 2)_{\text {corr }^{\text {d }}}{ }^{\text {d }}$ | $\Delta d^{\prime e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R | CABMOH, CABMOH01 | XD, rt | 2.526(3) | 1.263 | 1.263 | linear | 0 |  | 0.03 |
| R | refs 6-8 | ND, 20 K | 2.556(4) | 1.278(3) | 1.278 (3) | linear | 0 |  | 0.11 |
| A | BECHOG | XD, rt | 2.610(15) | [1.305] | $1.305(7)$ | linear | 0 |  | 0.18 |
| B | ROHTIR | XD, 200 K | 2.62 | 1.31 | [1.31] | linear | 0 |  | 0.19 |
| C | BEXROL02 | XD, 120 K | 2.629(4) | 1.31 | 1.31 | linear | 0 |  | 0.20 |
| D | LIJTED | XD, 128 K | 2.634(4) | [1.317] | [1.317] | linear | 0 |  | 0.21 |
| E | BEXROL | ND, 120 K | 2.635(2) | 1.317(1) | [1.317] | linear | 0 |  | 0.21 |
| F | ROHTIR | XD, 200 K | 2.64 | 1.32 | [1.32] | linear | 0 |  | 0.21 |
| G | YERLUC | XD, 200 K | 2.64 | 1.24 | 1.40 | $175^{\circ}$ | 0.08 |  | 0.13 |
| H | YERLOW | XD, 200 K | 2.64 | 1.19 | 1.45 | $179{ }^{\circ}$ | 0.13 |  | 0.08 |
| I | TEHNAV | ND, 100 K | 2.644(2) | $1.106(5)$ | 1.608(3) | $153.5(5)^{\circ}$ | 0.29 | 0.25 | 0.02 |
| J | BEXROL01 | XD, rt | 2.648(9) | 1.29(4) | 1.44(4) | $152(2)^{\circ}$ | 0.12 | 0.08 | 0.18 |
| K | PATGIA | XD, 193 K | 2.650 | 1.048 | 1.632 | $162.3^{\circ}$ | 0.31 | 0.29 | -0.06 |
| L | APYRDN | ND, rt | 2.698(8) | 1.17(2) | 1.52 | $177(2)^{\circ}$ | 0.17 |  | 0.08 |
| M | ROHTAJ | XD, 200 K | 2.70 | 1.08 | [1.62] ${ }^{f}$ | $175^{\circ}$ | 0.27 |  | -0.01 |
| N | PYCBZN01 | ND, rt | 2.737(3) | 1.086 (7) | 1.658(6) | $172(1)^{\circ}$ | 0.29 | 0.29 | 0.01 |
| P | CYPYFE | XD, rt | 2.74(2) | [1.01] ${ }^{\text {f }}$ | 1.86(1) | $144^{\circ}$ | 0.49 | 0.43 | -0.07 |
| Q | HDRZHO11 | ND, rt | 2.87 | 1.05 | 1.83 | $174{ }^{\circ}$ | 0.40 | 0.40 | 0 |

${ }^{a}$ This listing, while not exhaustive, is the result of a survey of homoconjugated or near-homoconjugated $\mathrm{ZN}-\mathrm{H}-\mathrm{NZ}$ bonds in the Cambridge Structural Data Base (CSDB). The code letters in the first column refer to Figure 4. The bonds are interpreted as centrosymmetric in A-C, E, F, and R. ${ }^{b}$ Reference code in the CSDB. ${ }^{c} z / 2=d^{\prime \prime}-(D / 2) .{ }^{d}(z / 2)_{\text {corr }}$ refers to $D_{\text {corr }}=d^{\prime}+d^{\prime \prime}$ in bonds where NHN $<175^{\circ}$. ${ }^{e} \Delta d^{\prime}=d^{\prime}$ expt $-d^{\prime}$ curve (Figure 4). ${ }^{f}$ Distance calculated from the NHN angle quoted.


Figure 4. Variation, with $D$, of the displacement of $z / 2$ of the H atom from the center of the $\mathrm{N} \cdots \mathrm{N}$ distance in homoconjugated (prosymmetric) $\mathrm{ZN}-\mathrm{H}-\mathrm{NZ}$ bonds. The curve represents the ab initio functional relationship expressed from (3) $+(4 \mathrm{a})$ and agrees, within its $\sigma \sim 0.02$ $\AA$ (the $\pm 1 \sigma$ bar), with Steiner's regression (1) $+(2)$ based on neutron diffraction results. For the experimental points A-R see Table 5 and text. The $D$ and $z / 2$ values for $\mathrm{I}, \mathrm{J}, \mathrm{K}$, and P (reported NHN $<175^{\circ}$ ) have been adjusted ( $\mathrm{I}^{\prime}, \mathrm{J}^{\prime}, \mathrm{K}^{\prime}$, and $\mathrm{P}^{\prime}$ ) assuming $D_{\text {corr }}=d^{\prime}+d^{\prime \prime}$.

Figure 4 shows clearly that the experimental point R and those with $D \geq 2.7 \AA$ ( $\mathrm{M}, \mathrm{N}, \mathrm{Q}$, and also $\mathrm{I}^{\prime}, \mathrm{K}^{\prime}, \mathrm{P}^{\prime}$ after adjusting the $D$ and $z / 2$ of I, K, P to $D_{\text {corr }}=d^{\prime}+d^{\prime \prime}$ ) are well accommodated by the ab initio curve generated by (3) + (4a), with a vertical error width $\pm 1 \sigma \sim \pm 0.02 \AA$; we recall that this curve is statistically equivalent to the curve that would be obtained from Steiner's regression (1) $+(2)$. However, for bonds with $D=$ $2.60-2.64 \AA$ and interpreted as linear symmetric (A-F; all except D presumed to have the H nucleus on an inversion center), the $z / 2$ values deviate from the curve value by $\sim 0.2 \AA$. For the remaining points $\mathrm{G}, \mathrm{H}, \mathrm{J}$, and L , the $z / 2$ values deviate from the curve by lesser amounts (Table 5). In the face of these experimental values, which converge fitfully toward symmetry as $D$ decreases, to what degree can one have confidence in the applicability of the ab initio curve to short, homoconjugated $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds in crystals?

Although the plot of Figure 4 looks confusing, we can show that, acknowledging the obvious differences (isolated static
"dimers" at $T \sim 0$ vs $\mathrm{ZN}-\mathrm{H}-\mathrm{NZ}$ fragments embedded in crystals at finite temperatures) and the $\sigma$ fit of the ab initio curve, the curve and the experimental points are in fact not incompatible if the following is kept in mind. (i) The positions of the N atoms (i.e., the $D$ value) are at least as (X-rays) or equally (neutrons) accurate as the putative position of the H atom. (ii) The above $z / 2$ deviations of $\sim 0.2 \AA$ from the curve values for A-F correspond to $U(\mathrm{H}) \| \sim 0.05 \AA^{2}$, a value of a magnitude comparable to the $U(\mathrm{H})_{\text {iso }}$ or $U(\mathrm{H}) \|$ quoted for the actual structure refinements. (iii) As is well-known, ${ }^{24 b, 25}$ when, in a structure refinement, the crystal symmetry calls for placing the H atom or the proton in an $\mathrm{ZX}-\mathrm{H}-\mathrm{XZ}$ bond on a crystallographic symmetry element, ${ }^{26}$ the possibility that a static or dynamic disorder is present must be considered. Whether the evidence is interpreted in favor of the H atom or the proton as on or off the symmetry element will depend on the kind and the resolution of the diffraction data, and on the temperature of the experiment (ambient or low; single vs two or more temperatures). While the interpretation of the evidence may depend critically on the resolution, this fact has not always received due attention.

The effect of the experimental conditions on the interpretation of the nature of the $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bond in the perchlorate of the [(quinuclidin-3-one) $\left.)_{2} \mathrm{H}\right]^{+}$cation has been investigated by Rozière et al. ${ }^{24}$ (Table 5). The short bond in the 120 K structure, determined both by neutron (E, BEXROL) and X-ray (C, BEXROL02) diffraction, is interpreted as symmetric, with the hydrogen on an inversion center. In the room-temperature structure (X-rays, J, BEXROL01) the difference Fourier map suggests that the bond is asymmetric and bent (cf. $\mathrm{J}^{\prime}$ ), with the H atom in 2-fold disorder. The coalescence of the two maxima in the difference map on cooling (cf. the dashed line from J to C as well as to E in Figure 4), without significant change in the $N \cdots N$ distance, seems to be consistent with dynamic disorder, the increased thermal motion at room-temperature forcing the H atom off the $\mathrm{N} \cdots \mathrm{N}$ line. However, although on the face of it the evidence for BEXROL and BEXROL02 appears to be consistent with a centrosymmetric bond, this interpretation is, to the authors' own admission, not unassailable. Indeed, the rms displacement of the proton along the $\mathrm{N} \cdots \mathrm{N}$ line, which in BEXROL is stated to be $\sim 0.19 \AA$, when taken
as $z / 2$ would place H on the ab initio curve (consistent with the $d^{\prime} \sim 1.11 \AA$ appropriate to the observed $D=2.63 \AA$ and $z / 2 \sim$ $0.20 \AA$ ) and thus argue for an asymmetric quasilinear $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bond.

Similar consideration would apply to A, B, D, and F, all interpreted as symmetric, and to the asymmetric G, H, and L, which inclusion of experimental $U(\mathrm{H})$ might raise to the ab initio curve. Without re-refining the original diffraction data (and perhaps not even then) with the H atom placed at the $z / 2$ distance estimated from the ab initio curve, it is difficult to say whether in the individual cases (and excepting R ) the experimental evidence for the $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds put forward as linear in fact supports or militates against the expectation from the $z / 2$ vs $D$ correlation of Figure 4. Nonetheless, with this correlation now available and coherent up to $D \sim 3.5 \AA$, it would seem worthwhile to use it as a guide in future structure determinations involving such bonds and to design the experiments with a view to corroboration or otherwise of the present conclusions from the ab initio calculations. Reference to this correlation might also avoid assuming inappropriate $\mathrm{N}-\mathrm{H}$ distances when describing the geometry of $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds in cases where direct evidence of the H position is doubtful or lacking (cf. the "standard" neutron $\mathrm{N}-\mathrm{H}$ bond length of $1.02 \AA$, which would be applicable only at $D>4 \AA$, i.e., at the margins of hydrogen bonding, cf. for example NOZQUO, SAXBEY).

The plot of Figure 4 is applicable to linear or near-linear $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds and does not lend itself to direct comparisons where the bonds are significantly bent. The above adjustment of the $D\left(\right.$ NHN $\left.<175^{\circ}\right)$ values to $D_{\text {corr }}=d^{\prime}+d^{\prime \prime}$ would be justified only if the angular dependence of the H -bond strength was small, but the fact that the adjusted points $\mathrm{I}^{\prime}, \mathrm{K}^{\prime}$, and $\mathrm{P}^{\prime}$ (though not $\mathrm{J}^{\prime}$ ) are closer to the ab initio curve than the corresponding unadjusted points would seem to suggest that the angular dependence is not large.

Position of the Bond Critical Point. The variation of the position of the BCP $X^{\prime}$ in the $\mathrm{N}_{\mathrm{d}}-\mathrm{H}$ bond and of the BCP $\mathrm{X}^{\prime \prime}$ in the $H \cdots N_{a}$ bond in the optimized structures of $S$ can be displayed in various ways, all of which show that the position of $\mathrm{X}^{\prime}$ is strongly correlated with the position of its conjugate $\mathrm{X}^{\prime \prime}$ as well as with $d^{\prime}, d^{\prime \prime}$, and $D$. When the $\mathrm{N}_{\mathrm{d}} \cdots \mathrm{X}^{\prime}=x^{\prime}$ and $\mathrm{X}^{\prime \prime} \cdots \mathrm{N}_{\mathrm{a}}=x^{\prime \prime}$ distances are plotted against $D$ (Figure 2B), the means $\left(x^{\prime}+x^{\prime \prime}\right) / 2$ can be represented to a high degree of correlation by

$$
\begin{equation*}
\left(x^{\prime}+x^{\prime \prime}\right) / 2=0.479(17)+0.197(6) D \tag{5}
\end{equation*}
$$

( $r^{2}=0.952, \sigma=0.011 \AA \sim 5 \%$ of range). The resulting regression line, which relates the $N_{d} \cdots X^{\prime}$ and $X^{\prime \prime} \cdots N_{a}$ branches of the plot, is analogous to the rectilinear diameter of $d^{\prime}, d^{\prime \prime}$ in Figure 2A. For the symmetric situation at $D_{\min }=2.49(20) \AA$, $x^{\prime}=x^{\prime \prime}=x_{\text {sym }}$ would be $0.97(3) \AA$. The $x^{\prime}$ and $x^{\prime \prime}$ parameters are thus clearly correlated and closely mimic $d^{\prime}$ and $d^{\prime \prime}$. This is further confirmed by regressing $x^{\prime}$ on $x^{\prime \prime}$ according to (1) as the model function:

$$
\begin{align*}
x^{\prime}=x_{0}-\beta \ln \left\{1-\exp \left[\left(x_{0}-x^{\prime \prime}\right) / \beta\right]\right\}, x_{0} & =0.734(6) \AA \\
\beta & =0.356(10) \AA \tag{6}
\end{align*}
$$

( $r^{2}=0.935 \AA, \sigma=0.010 \AA \sim 7 \%$ of range, $x_{\text {sym }}=0.981(13)$
$\AA$; Figure 1A). Here $x_{0}=x_{\text {sym }}-\beta \ln 2$, the limiting distance, represents the effective radius of the N atom in the symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bond, from which $r(\mathrm{H})=0.27(2) \AA$ is the associated radius of the H atom in the bond.

The positions of $\mathrm{X}^{\prime}$ and $\mathrm{X}^{\prime \prime}$ in their respective bonds are plotted in Figure 2B: combining (5) and (6) yields a two-valued
function of $D$ implicit in $x^{\prime}$ and $x^{\prime \prime}$. For $D>3 \AA$ (i.e., $d^{\prime \prime}>2$ $\AA$ ), the $x^{\prime}$ and the $x^{\prime} / d^{\prime}$ ratio are essentially constant, whereas $x^{\prime \prime}$ increases with $d^{\prime \prime}$.

The corresponding regressions for the MP2 set (Figures 1B and 2D) yield

$$
\begin{equation*}
\left(x^{\prime}+x^{\prime \prime}\right) / 2=0.482(18)+0.192 D \tag{5a}
\end{equation*}
$$

( $r^{2}=0.983, \sigma=0.009 \AA \sim 5 \%$ of range ); $x_{\text {sym }}=0.967 \AA$ for $D_{\text {min }}=2.524 \AA$ from (4a) and

$$
\begin{equation*}
x_{0}=0.741(16) \AA, \beta=0.330(24) \AA, x_{\mathrm{sym}}=0.970(33) \AA \tag{6a}
\end{equation*}
$$

$\left(r^{2}=0.948 \AA, \sigma=0.018 \AA \sim 8 \%\right.$ of range, $D_{\text {min }}=2.542(86)$ $\AA$, from (5a)). The radius $r(\mathrm{H})$ of the H atom in the symmetric bond is estimated from (4a)+(6a) as $0.29(3) \AA$, in agreement with the above HF value of $0.27(2) \AA$.

The difference between the $D_{\min }(\mathrm{HF})$ values calculated from (4) and from $(5)+(6)$, respectively, is well within the joint standard error of the two values, as is the difference between the $D_{\min }($ MP2 ) values calculated from (4a) and from (5a) + (6a), respectively.

## Electron Densities

Electron Density $\rho_{\mathbf{c}}$ at the BCP: The HF Set. While the above functional relationships between the internuclear distances in $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds derive from Pauling's (semiempirical) bondorder equation, no such guiding principle appears to have been formulated for representing the variation of the electron density $\rho_{\mathrm{c}}$ at the BCP with $d$. Descriptions of the observed $d, \rho_{\mathrm{c}}$ dependence are therefore of necessity empirical, and their relative merits have to be judged largely on statistical criteria. A plot of $\rho^{\prime}=\rho_{\mathrm{c}}\left(\mathrm{X}^{\prime}\right)$ at the BCP $\mathrm{X}^{\prime}$ and $\rho^{\prime \prime}=\rho_{\mathrm{c}}\left(\mathrm{X}^{\prime \prime}\right)$ at the BCP $\mathrm{X}^{\prime \prime}$ against the corresponding $d^{\prime}$ and $d^{\prime \prime}$ suggests that the variation of $\rho_{\mathrm{c}}$, which spans almost 2 orders of magnitude, is continuous and representable by a simple model function, for which $\rho^{\prime \prime} \rightarrow 0$ as $d^{\prime \prime} \rightarrow \infty$.

The two simplest such functions are the power function

$$
\begin{equation*}
\rho_{\mathrm{c}}=a d^{-b} \tag{7}
\end{equation*}
$$

and the exponential function

$$
\begin{equation*}
\rho_{\mathrm{c}}=a \exp (-b d) \tag{8}
\end{equation*}
$$

Regressions (not linearized) according to (7) and (8) yielded the parameters listed in Table 6. For the HF set (R71 and R81), the $r^{2}$ and $\sigma$ values indicate that the $d, \rho_{\mathrm{c}}$ are highly correlated. The adequacy of representing this data set by either model function might thus not be called in question and the matter might rest there. However, further inspection reveals that the residuals $\Delta \rho_{\mathrm{c}}=\rho_{\mathrm{c}}$ (tabulated) $-\rho_{\mathrm{c}}$ (regression) are not distributed uniformly. The $\Delta \rho^{\prime}$ in the $d^{\prime}, \rho^{\prime}$ subset range approximately from -0.013 to 0.039 au for R71, and approximately from -0.018 to 0.031 au for R81, whereas $\Delta \rho^{\prime \prime}$ in the $d^{\prime \prime}, \rho^{\prime \prime}$ subset range approximately from -0.008 to 0.010 au for R71 and approximately from -0.008 to 0.014 au for R 81 ; the nonuniformity of the error variance is shown, in logarithmic presentation, in Figures 5A,C. Thus, the adequacy of representing the HF $d, \rho_{\mathrm{c}}$ data set by (7) or (8) cannot be assessed on the strength of $r^{2}$ and $\sigma$ alone.

The uniformity of the error variance improves when the nonlinear regression functions (7) and (8) are replaced by the

TABLE 6: Regressions of the Electron Density $\rho_{\mathrm{c}}$ at the BCP on the $\mathbf{N}$-H Distances ${ }^{a}$

|  | data set ${ }^{\text {b }}$ | n | $a$ | $b$ | $r^{2}$ | $\sigma$ | \% ${ }^{\text {c }}$ | $r(a, b)^{d}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Power Function, Eq 7 |  |  |  |  |  |  |  |  |
| R71 | HF, all | 124 | 0.345(2) | 3.65(7) | 0.992 | 0.0127 | 3.6 | 0.396 |
| R72 | HF, $d^{\prime}, \rho^{\prime}$ only | 63 | 0.340 (3) | 2.90(21) | 0.768 | 0.0159 | 13.6 | 0.623 |
| R73 | HF, $d^{\prime \prime}$, $\rho^{\prime \prime}$ only | $61^{e}$ | 0.441 (20) | 4.25(9) | 0.982 | 0.0029 | 3.7 | 0.981 |
| R74 | MP2, all | 31 | $0.355(5)$ | 3.50(10) | 0.990 | 0.0107 | 3.3 | 0.653 |
| R75 | MP2, $d^{\prime}, \rho^{\prime}+$ sym | 19 | 0.355(6) | 3.46(18) | 0.967 | 0.0139 | 7.5 | 0.707 |
| R76 | MP2, $d^{\prime \prime}, \rho^{\prime \prime}+$ sym | $17^{e}$ | 0.342(15) | 3.46(15) | 0.990 | 0.0058 | 4.0 | 0.946 |
| R77 | MP2, all - sym | 26 | $0.355(5)$ | 3.42(12) | 0.991 | 0.0117 | 3.6 | 0.608 |
| R78 | MP2, $d^{\prime}, \rho^{\prime}$ - sym | 14 | 0.348(6) | 3.08 (24) | 0.942 | 0.0128 | 8.3 | 0.717 |
| R79 | MP2, $d^{\prime \prime}, \rho^{\prime \prime}-$ sym | $12^{e}$ | $0.385(35)$ | 3.71 (23) | 0.982 | 0.0059 | 4.7 | 0.964 |
| Exponential Function, Eq 8 |  |  |  |  |  |  |  |  |
| R81 | HF, all | 124 | 2.69 (32) | 2.82(5) | 0.993 | 0.0115 | 3.3 | 0.997 |
| R82 | HF, $d^{\prime}, \rho^{\prime}$ only | 63 | 5.59(12) | 2.80(2) | 0.770 | 0.0150 | 12.8 | 1.000 |
| R83 | HF, $d^{\prime \prime}$, $\rho^{\prime \prime}$ only | $61^{e}$ | 2.86(25) | 2.41(5) | 0.982 | 0.0028 | 3.6 | 0.995 |
| R84 | MP2, all | 31 | 6.28(62) | 2.90(9) | 0.999 | 0.0110 | 3.4 | 0.996 |
| R85 | MP2, $d^{\prime}, \rho^{\prime}+$ sym | 19 | 7.78(120) | 3.10(14) | 0.972 | 0.0127 | 6.9 | 0.997 |
| R86 | MP2, $d^{\prime \prime}, \rho^{\prime \prime}+$ sym | $17^{e}$ | 2.76 (24) | 2.30(7) | 0.996 | 0.0036 | 2.5 | 0.995 |
| R87 | MP2, all - sym | 26 | 5.46 (55) | 2.76 (9) | 0.993 | 0.0108 | 3.3 | 0.995 |
| R88 | MP2, $d^{\prime}, \rho^{\prime}$ - sym | 14 | 6.00(14) | 2.85(21) | 0.945 | 0.0125 | 8.1 | 0.999 |
| R89 | MP2, $d^{\prime \prime}, \rho^{\prime \prime}$ - sym | $12^{e}$ | 2.87(43) | 2.32(10) | 0.993 | 0.0038 | 3.1 | 0.995 |

[^2] range. ${ }^{d}$ Correlation coefficient for the regression coefficients $a$ and $b .{ }^{e}$ Data set does not include $\mathrm{NH}_{4}^{+}(66)$ and $\mathrm{F}_{3} \mathrm{NH}^{+}(67)$.
logarithmic versions of these functions
\[

$$
\begin{equation*}
\ln \rho_{\mathrm{c}}=\ln a-b \ln d \tag{7a}
\end{equation*}
$$

\]

( $r^{2}=0.994, a=0.354, b=3.949, \sigma=0.0974 \sim 2.5 \%$ of the $\ln \rho_{\mathrm{c}}$ range) and

$$
\begin{equation*}
\ln \rho_{\mathrm{c}}=\ln a-b d \tag{8a}
\end{equation*}
$$

( $r^{2}=0.994, a=4.610, b=2.644, \sigma=0.0980 \sim 2.6 \%$ of range). Since the ranges of $\Delta \rho^{\prime}$ and $\Delta \rho^{\prime \prime}$ in R71 and R81 correspond roughly to the same fraction (5-10\%) of the respective $\rho^{\prime}$ and $\rho^{\prime \prime}$ values, it can be argued that the logarithmic functions (7a) and (8a) are more appropriate regression functions than (7) and (8). In any event, it is clear that the inherent scatter of the $d^{\prime}, \rho^{\prime}$ and $d^{\prime \prime}, \rho^{\prime \prime}$ data points is the factor that limits further improvement in the fit by any single monotonic regression function.

The absence of data points in the interval between the two subsets raises the following question: Is representing the $d, \rho_{\mathrm{c}}$ correlation by a single smooth, continuous function, such as (7) or (8), in fact warranted by the data set, or is the correlation represented better by two separate such functions, one for each subset? In Table 6, the power regression R71 is contrasted with R72 and R73, and the exponential regression R81 is contrasted with R82 and R83. From the $\sigma$ values for R73 and R83, it is evident that the $d^{\prime \prime}, \rho^{\prime \prime}$ correlation is represented better by a separate regression, while for the $d^{\prime}, \rho^{\prime}$ correlation the fidelity of the separate regression is inferior to that achieved by a single global function R71 or R81. Statistical tests on the sums of squares of the residuals show that the two separate regressions taken jointly fit the data set significantly better than the corresponding single-function regressions, and this is seen directly on rectifying the regression lines in Figure 5, where R71-R73 and R81-R83 are presented as logarithmic transformations $\ln$ R71 to R73 and ln R81 to ln R83. Furthermore, the uniformity of distribution of the residuals is significantly improved when two functions are used (Figure 6).

Additional improvement in the goodness of fit and the uniformity of distribution of the residuals results when, instead of the direct regressions (7) and (8) (or their logarithmic transformations), logarithmic regressions are used for the two subsets. This is analogous to what is observed for the single-


Figure 5. Fitting the $\mathrm{HF} d, \rho_{\mathrm{c}}$ data set to a single (A) and to two (B) power functions and to a single (C) and to two (D) exponential functions. The rectified correlation lines represent the logarithmically transformed correlation functions R71-R73 and R81-R83, respectively, of Table 6.
function logarithmic regressions (7a) and (8a) and supports the view that the logarithmic regressions represent the $d, \rho_{c}$ correlation more faithfully.

The single-function representation can be rejected, at a $>99.5 \%$ significance level, in favor of the corresponding twofunction representation for both the logarithmically transformed regressions of Table 6 and for the logarithmic regressions, power as well as exponential. Thus, of the representations examined above, the two-function logarithmic regressions represent the $\mathrm{HF} d, \rho_{c}$ correlation best. It should be noted, however, that while the single-function power regression R71 overestimates the $\rho_{c}$ values for large $d$ (Figure 5A), the corresponding exponential R81 regression underestimates them (Figure 5C). This would suggest that a hybrid model function which combines (7) and (8), $\rho_{\mathrm{c}}=\alpha(7)+(1-\alpha)(8)$, or the corresponding logarithmically transformed functions, might improve the fit overall and specifically for $d^{\prime \prime}, \rho^{\prime \prime}$. Indeed, the sum $\rho_{\mathrm{c}}=(\ln \mathrm{R} 71+\ln$ R81)/2 (i.e., $\alpha=0.5$ ) gives a lower $\sigma$ value than either $\ln \mathrm{R} 71$ or $\ln$ R81, and optimizing $\alpha$ might produce a fit better still. However, vis-à-vis the simple functions such composite model functions


Figure 6. Distribution of the residuals when the correlation of $\rho_{c}$ and $d$ for the HF set is represented by a single (A) and by two (B) power functions and by a single (C) and by two (C) exponential functions. The residuals $\Delta \ln \rho_{\mathrm{c}}=\ln \rho_{\mathrm{c}}($ tabulated $)-\ln \rho_{\mathrm{c}}$ (calculated) correspond to Figure 5, i.e., to logarithmically transformed regression equations R71-R73 and R81-R83, respectively, of Table 6.
appear somewhat contrived and have not been investigated further.

Regressing $\rho_{\mathrm{c}}$ on $d$ does not take account of the pairwise relationship of $d^{\prime}$ to $d^{\prime \prime}$ and of $\rho^{\prime}$ to $\rho^{\prime \prime}$. This neglect of the conjugate nature of $d^{\prime}, d^{\prime \prime}$ and $\rho^{\prime}, \rho^{\prime \prime}$ is remedied by taking the ratios $d^{\prime} / d^{\prime \prime}$ and $\rho^{\prime} / \rho^{\prime \prime}$. For the single functions

$$
\begin{align*}
& \ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=b \ln \left(d^{\prime \prime} / d^{\prime}\right)  \tag{7b}\\
& \ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=b\left(d^{\prime \prime}-d^{\prime}\right) \tag{8b}
\end{align*}
$$

These ratios contain only one parameter to be evaluated and implicitly satisfy the requirement that for symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds (not present in the HF set) $\rho^{\prime} / \rho^{\prime \prime}=d^{\prime \prime} / d^{\prime}=1$; i.e., the regression line is constrained to pass through the origin. However, for two regression functions

$$
\begin{gather*}
\ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=\ln \left(a^{\prime} / a^{\prime \prime}\right)+\left(b^{\prime \prime}-b^{\prime}\right) \ln d^{\prime \prime}+b^{\prime} \ln \left(d^{\prime \prime} / d^{\prime}\right)  \tag{7c}\\
\ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=\ln \left(a^{\prime} / a^{\prime \prime}\right)+\left(b^{\prime \prime}-b^{\prime}\right) d^{\prime \prime}+b^{\prime}\left(d^{\prime \prime}-d^{\prime}\right) \tag{8c}
\end{gather*}
$$

and the regression plane in the three-dimensional plot does not pass through the origin.

This and the goodness of fit of (7c) and (8c) can be used as another test of the single- vs two-function hypothesis. Thus, for (7b) we obtain by iteration $b=3.925, \sigma(7 \mathrm{~b})=0.1119$ $\sim 4.0 \%$ of range; for ( 8 b ), $b=2.674, \sigma(8 \mathrm{~b})=0.1187 \sim 4.2 \%$ of range. For the two-function model, regressing $\rho^{\prime} / \rho^{\prime \prime}$ on $d^{\prime \prime} / d^{\prime}$ and $d^{\prime \prime}$ according to (7c) yields

$$
\begin{array}{r}
\ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=-0.57(57)+2.56(99) \ln d^{\prime \prime}+ \\
2.15(82) \ln \left(d^{\prime \prime} / d^{\prime}\right) \tag{7d}
\end{array}
$$

where $r^{2}=0.985, \sigma(7 \mathrm{~d})=0.0914 \sim 3.3 \%$ of range; similarly, $\rho^{\prime} / \rho^{\prime \prime}$ regressed on $d^{\prime \prime}-d^{\prime}$ and $d^{\prime \prime}$ according to (8c) yields

$$
\begin{equation*}
\ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=2.20(120)-1.75(73) d^{\prime \prime}+4.00(65)\left(d^{\prime \prime}-d^{\prime}\right) \tag{8d}
\end{equation*}
$$

where $r^{2}=0.987, \sigma(8 \mathrm{~d})=0.0855 \sim 3.1 \%$ of range. Thus, while the absolute terms in (7d) and (8d) are nonzero, their error estimates do not preclude the possibility that the regression planes do pass through the origin and the test for the HF set is inconclusive.

For $d^{\prime}=d^{\prime \prime}$, i.e., for the symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds and with $d_{\text {sym }}=1.249(9) \AA$ from (4), the two-function representation implies that $\rho^{\prime} / \rho^{\prime \prime}=1.00(2)$ for the power fit and 1.07(2) for the exponential fit. These values would be 1.04 and 1.20 , respectively, if the $a$ and $b$ were taken from Table 6 . Conversely, for $d^{\prime}=d^{\prime \prime}$ and $\rho^{\prime}=\rho^{\prime \prime}$ simultaneously, $d_{\text {sym }}$ would be $1.25 \AA$ for (7d) and $1.30 \AA$ for (8d), values equivalent within the overall statistics.

Electron Density $\rho_{c}$ at the BCP: The MP2 Set. The above results for the HF set can be compared with an analogous treatment of the MP2 set. While the $d$ and $\rho_{\mathrm{c}}$ values in the MP2 set might be expected to be more realistic, their number is considerably smaller and their distribution over the $d^{\prime}$ and $d^{\prime \prime}$ ranges is not as uniform as in the HF set. However, they afford an opportunity to examine the effect of inclusion of the symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds on the correlations.

The representations R74 and R84 of the $d, \rho_{\mathrm{c}}$ data set (including the symmetric bonds) by a single function each are statistically equivalent, as are the logarithmic transformations In R74 and ln R84 of these regression equations and also the logarithmic regressions of $\ln \rho_{\mathrm{c}}$ on $\ln d$ and of $\ln \rho_{\mathrm{c}}$ on $d$ (cf. (7a) and (8a)); of these, the $\ln \rho_{c}$ on $d$ regression yields the most uniform distribution of the residuals. However, comparing ln R74 with the corresponding logarithmic regression $\ln \rho_{\mathrm{c}}$ on $\ln d$ reveals that these two sets are not statistically equivalent, and similarly for $\ln$ R84 and the logarithmic regression $\ln \rho_{\mathrm{c}}$ on $d$, the logarithmic regression being significantly better.

In comparing the MP2 results with those for the HF $d, \rho_{\mathrm{c}}$ set, it is appropriate to exclude the symmetric cases from the MP2 set (sets R77 and R87). Here, the error variance ratio for the logarithmically transformed R81 and R87 equations (ln R81 and In R87) shows that these two regressions are statistically equivalent (at the $95 \%$ significance level), whereas the $\ln$ R71 and $\ln$ R77 are not; the logarithmic regressions $\ln \rho_{\mathrm{c}}$ on $\ln d$ for the two sets are equivalent, but the corresponding regressions $\ln \rho_{\mathrm{c}}$ on $d$ for the two sets are not. For the $\mathrm{H} \cdots \mathrm{N}_{\mathrm{a}}$ end of the $\rho_{\mathrm{c}}$ vs $d$ correlation, testing the MP2 $d^{\prime \prime}, \rho^{\prime \prime}$ subset against the HF $d^{\prime \prime}, \rho^{\prime \prime}$ subset (R73 and R79, and R83 and R89) shows statistical equivalence of $\ln$ R83 vs $\ln$ R89 but not of $\ln R 73$ vs $\ln R 79$. The logarithmic regressions of $\ln \rho^{\prime \prime}$ on $\ln d^{\prime \prime}$, and of $\ln \rho^{\prime \prime}$ on $d^{\prime \prime}$, are statistically equivalent for both the HF and the MP2 sets.

Turning now to the effect of inclusion of the symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds (absent in the HF set), we compare the singlefunction regressions R74 with R77 and R84 with R87 (Table 6). Within the data range, the maximum difference between R74 and R77 is well within a single $\sigma$ of either regression, and similarly for R84 and R87. The logarithmically transformed regression equations $\ln \mathrm{R} 74$ and $\ln \mathrm{R} 77$ are statistically equivalent but ln R84 and ln R87 are not; the logarithmic regressions, however, are equivalent for both the power and the exponential model functions. There is thus no strong evidence that the MP2 set with the symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds included is statistically different from the MP2 set with the symmetric cases omitted.

The question still remains whether the two-function model represents our MP2 data better than the single-function regressions. For the MP2 set with the symmetric cases excluded (R78 and R79, and R88 and R89) the comparison would mirror that for the HF set, but for the inclusive MP2 set (R75 and R76, and R85 and R86) the comparison is more problematic in that the two sets being compared are not mutually exclusive (Table 6 ). The results for the logarithmically transformed regression equations of Table 6 (ln R77 vs $\ln$ R78 and $\ln$ R79; ln R87 vs


Figure 7. Distribution of the residuals when the correlation of $\rho_{c}$ and $d$ for the MP2 set (excluding the symmetric bonds) is represented by a single (A) and by two (B) power functions and by a single (C) and by two (C) exponential functions. The residuals $\Delta \ln \rho_{\mathrm{c}}=\ln$ $\rho_{\mathrm{c}}$ (tabulated) $-\ln \rho_{\mathrm{c}}$ (calculated) correspond to Figure 5, i.e., to logarithmically transformed regression equations R77-R79 and R87R89, respectively, of Table 6 and can be compared to those for the HF set in Figure 6.
ln R88 and ln R89) are presented in Figure 7. The distributions of the residuals are similar to those for the HF set (Figure 6). The $\rho^{\prime \prime}$ values again are underestimated in the single-function power regression and overestimated in the single-function exponential regression, i.e., this feature is retained in the MP2 optimizations. The two-function representations are significantly better than the corresponding single-function best fits at a $>99.5 \%$ significance level.

The single-function logarithmic regressions with the symmetric cases omitted are better fits than the corresponding logarithmically transformed power or exponential regressions at a $>99.5 \%$ significance level but can be rejected in favor of the two-function logarithmic regressions at the same significance level. However, the separate logarithmic regressions for $d^{\prime}, \rho^{\prime}$ and $d^{\prime \prime}, \rho^{\prime \prime}$ are statistically equivalent to the corresponding logarithmically transformed regressions (Figures 7B,D), with slightly smaller residuals. The best representation of the $d, \rho_{\mathrm{c}}$ correlation for the (MP2 - sym) set is

$$
\begin{aligned}
\ln \rho^{\prime}=-1.060-3.123 \ln d^{\prime}, r^{2}= & 0.925, \sigma= \\
0.0559 & \sim 9 \% \text { of range }
\end{aligned}
$$

$$
\ln \rho^{\prime \prime}=-0.640-4.448 \ln d^{\prime \prime}, r^{2}=0.994, \sigma=
$$

$$
0.0790 \sim 3 \% \text { of range }(7 \mathrm{e})
$$

for the power, and

$$
\begin{aligned}
& \ln \rho^{\prime}=1.810-2.875 d^{\prime}, r^{2}=0.930, \sigma= \\
& 0.0539 \sim 9 \% \text { of range }
\end{aligned}
$$

$$
\begin{aligned}
\ln \rho^{\prime \prime}=1.145-2.387 d^{\prime \prime}, r^{2}= & 0.997, \sigma= \\
0.0381 & \sim 1.5 \% \text { of range }(8 \mathrm{e})
\end{aligned}
$$

for the exponential regression.
These two representations are statistically equivalent to the logarithmically transformed regressions of the $\rho^{\prime}+\rho_{\text {sym }}$ subset on the $d^{\prime}+d_{\text {sym }}$ subset (ln R78 and $\ln$ R88) and of the $\rho^{\prime \prime}+$ $\rho_{\text {sym }}$ subset on the $d^{\prime \prime}+d_{\text {sym }}$ subset (ln R76 and ln R86), and in turn equivalent to the corresponding logarithmic regressions
of these subsets. Overall, the best representation of the MP2 $d, \rho_{\mathrm{c}}$ correlation is

$$
\ln \rho^{\prime}=2.106-3.154 d^{\prime}, r^{2}=0.977, \sigma=
$$

$$
0.0496 \sim 6 \% \text { of range }
$$

$\ln \rho=1.116-2.374 d^{\prime \prime}, r^{2}=0.998 . \sigma=$ $0.0526 \sim 2 \%$ of range, ( 8 f )
with the two regression lines intersecting at $d \sim 1.27 \AA$, i.e., close to the $d_{\text {sym }}=1.26(2) \AA$ value from eq 4 a .

As discussed above, analysis of the $\rho^{\prime} / \rho^{\prime \prime}$ vs $d^{\prime \prime} / d^{\prime}$ correlation can be used to decide between the single- and the two-function models. While for the HF set the result was inconclusive, it was thought that a clearer picture might emerge for the MP2 set, despite the smallness of the latter. Iterative solution to (7b) gave $b=3.897, r^{2}=0.996, \sigma=0.0887 \sim 2.6 \%$ of range ( $r^{2}=0.992, \sigma=0.1061$ with the symmetric values excluded); and to ( 8 b ), $b=2.539, r^{2}=0.997, \sigma=0.0613 \sim 2 \%$ of range ( $r^{2}=0.995, \sigma=0.0740$ with the symmetric cases excluded). The power regression is statistically equivalent, at the $95 \%$ significance level, to that for the HF set, but the exponential is not.

Regression according to (7c) resulted in

$$
\begin{array}{r}
\ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=-0.74(20)+3.16(48) \ln d^{\prime \prime}+ \\
1.65(36) \ln \left(d^{\prime \prime} / d^{\prime}\right) \tag{7~g}
\end{array}
$$

$r^{2}=0.999,0=0.0393 \sim 1 \%$ of range; and in
$\ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=-0.63(46)+2.46(90) \ln d^{\prime}+$

$$
\begin{equation*}
2.24(72) \ln \left(d^{\prime \prime} / d^{\prime}\right) \tag{7h}
\end{equation*}
$$

$r^{2}=0.999, \sigma=0.0454 \sim 1.5 \%$ of range, with the symmetric cases excluded. Similarly, regression according to (8c) gave

$$
\begin{equation*}
\ln \left(\rho^{\prime} / \rho^{\prime \prime}\right)=1.62(42)-1.29(28) d^{\prime \prime}+3.57(23)\left(d^{\prime \prime}-d^{\prime}\right) \tag{8~g}
\end{equation*}
$$

$$
r^{2}=0.999, \sigma=0.0380 \sim 1 \% \text { of range; and }
$$

$$
\begin{equation*}
\left(\rho^{\prime} / \rho^{\prime \prime}\right)=2.33(97)-1.91(60) d^{\prime \prime}+4.12(53)\left(d^{\prime \prime}-d^{\prime}\right) \tag{8h}
\end{equation*}
$$

$r^{2}=0.998, \sigma=0.0436 \sim 1.5 \%$ of range, with the symmetric cases excluded. Consideration of the error estimates on the absolute terms shows that the regression planes for the full MP2 set, $(7 \mathrm{~g})$ and $(8 \mathrm{~g})$, do not pass through the origin; i.e., these representations clearly favor the two-function model; also, their $\sigma$ values are the lowest.

Neither (7g) nor (7h) is statistically equivalent to the corresponding HF regression (7d), and similarly neither ( 8 g ) nor ( 8 h ) is statistically equivalent to ( 8 d ).

Electron Density $\rho_{c}$ at the BCP: Conclusions. So far, we have not considered the question of preference between the power and the exponential regression. In most of the power/ exponential regression pairs the $r^{2}$ and $\sigma$ values are similar. However, because of the nonlinearity of (7) and (8), statistical equivalence of the two regressions in any given case cannot be established from the $r^{2}$ and $\sigma$ values alone, nor by any simple statistical decision rule. Generally, and on the face of it, the exponential model function generates somewhat better figures of merit, but an elaboration of more stringent comparison criteria is not contemplated here. The suitability and relative merits of the logarithmic model functions (7a) and (8a) have been investigated by Alkorta et al., ${ }^{28}$ who examined ab initio $d^{\prime}, \rho_{c}$ values in $24 \mathrm{~N}-\mathrm{H}$ bonds and also in a variety of other $\mathrm{X}-\mathrm{H}$ bonds, $\mathrm{X}=\mathrm{H}, \mathrm{C}, \mathrm{O}, \mathrm{F}, \mathrm{Cl}, \mathrm{Br}$. While the differences in the
composition of the $\mathrm{N}-\mathrm{H}$ data sets preclude a detailed comparison of the Alkorta regressions with ours, the regression coefficients in (7a) and (8a) are quite similar to those in the Alkorta study, the individual differences not exceeding $\sim 2 \%$ of the respective values except for $b$ in (7a), for which the difference was $\sim 8 \%$. These authors, too, conclude that both model functions represent their data sets more or less equally adequately, though slightly better results were obtained with the exponential model. They also note that the exponential but not the power function is consistent with the exponential falloff of $\rho$ with distance in atoms and molecules. ${ }^{29}$

The approximate range of $\rho_{\mathrm{c}}$ is $0.007-0.35$ au in the HF sample and $0.01-0.33 \mathrm{au}$ in the MP2 set. The lower bound of $\rho^{\prime \prime}$ is of course zero, while the upper bound of $\rho^{\prime}$ would be that in externally unengaged $\mathrm{N}-\mathrm{H}$ bonds ( $\rho_{\mathrm{c}}$ in au):

|  | $\rho_{c}(\mathrm{MP} 2)$ | $\rho_{c}(\mathrm{HF})$ | difference | ratio |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{H}_{3} \mathrm{~N}$ | 0.349 | 0.358 | 0.009 | 1.026 |
| $\mathrm{H}_{3} \mathrm{NH}^{+} \cdots \mathrm{F}^{-}$ | 0.349 | 0.357 | 0.008 | 1.023 |
| $\mathrm{H}_{3} \mathrm{~N}^{\cdots} \cdot \mathrm{HNC}(\mathbf{3 6})$ | 0.348 | 0.357 | 0.009 | 1.026 |
| $\mathrm{H}_{3} \mathrm{NH}^{+} \cdots \mathrm{CN}^{-}$ | 0.348 | 0.357 | 0.009 | 1.026 |
| HNC | 0.341 | 0.354 | 0.013 | 1.038 |
| $\mathrm{H}_{4} \mathrm{~N}^{+}(\mathbf{6 6})$ | 0.333 | 0.349 | 0.016 | 1.048 |
| $\mathrm{~F}_{3} \mathrm{NH}^{+}(\mathbf{6 7})$ | 0.332 | 0.356 | 0.024 | 1.072 |

Indeed, the largest $\rho^{\prime}$ in the HF sample are grouped about $\rho^{\prime} \sim$ 0.35 au (mean of the $\rho^{\prime}>0.34$ au values in Table 2, 0.346(4) au ), and in the MP2 sample, about $\rho^{\prime} \sim 0.33$ au (mean of the $\rho^{\prime}>0.32$ au values in Table 4, 0.327(3) au). It appears unlikely that, in isolated model species, linear or near-linear $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bonds will be found with $d^{\prime}$ significantly below $0.98 \AA$ and $\rho^{\prime}$ significantly above the $\rho_{\mathrm{c}}$ values in the preceding table, regardless of purposeful manipulation. Even in pertinent real crystals at high pressure it is improbable that such bonds would exist, as shorter $\mathrm{N}-\mathrm{H}$ bonds would require longer $\mathrm{H} \cdots \mathrm{N}$ bonds, i.e., the donor species would be increasingly non-H-bonded; instead, stronger and bent H -bonds would be expected (solid ammonia at high pressure would be a case in point ${ }^{30}$ ). To place these $\rho_{\mathrm{c}}(\mathrm{NH})$ values in a larger context, we may contrast them with the $\rho_{\mathrm{c}}$ in a very strong covalent bond. In the $\mathrm{N}_{2}$ molecule $(d(\mathrm{~N}-\mathrm{N})=1.094 \AA$ from experiment $), \rho_{\mathrm{c}}(\mathrm{HF}) \sim 0.711$ au and $\rho_{\mathrm{c}}($ MP2 $) \sim 0.632 \mathrm{au}$. These values must be very close to the respective (unknown ?) upper $\rho$ bond in any type of chemical bond.

In sum, our examination leads to the conclusion that the $d, \rho_{\mathrm{c}}$ correlation is represented, both in the HF and the MP2 set, significantly better by two, rather than a single, power or exponential functions. It is conceivable that a single smooth, continuous model function exists that may represent the correlation equally well, but if it does, it has not yet been formulated.

On reflection, the apparent preference for the two-function model to represent the $d, \rho_{\mathrm{c}}$ correlation is not unexpected, given the difference in the nature of the $\mathrm{N}-\mathrm{H}$ and $\mathrm{H} \cdots \mathrm{N}$ bonds at large $\mathrm{N}_{\mathrm{d}} \cdots \mathrm{N}_{\mathrm{a}}$ separations. This difference decreases with decreasing $D$, but the character of the convergence toward the symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bond from opposite sides, the covalent and the H -bonded, is not the same, as reflected in the unequal dependence of $\rho^{\prime}$ on $d^{\prime}$ and of $\rho^{\prime \prime}$ on $d^{\prime \prime}$. Thus, for the twofunction model, $\rho^{\prime}=\rho^{\prime \prime}$ at $d_{\text {sym }}=d^{\prime}-d^{\prime \prime}$, but $\mathrm{d} \rho^{\prime} / \mathrm{d} d^{\prime} \neq \mathrm{d} \rho^{\prime \prime} /$ $\mathrm{d} d^{\prime \prime}$.
The HF set, on one hand, does not lend itself to investigation of smooth continuity of the $d, \rho_{\mathrm{c}}$ correlation, for the sizable gap it contains between $d^{\prime}(\mathrm{HF})_{\max } \sim 1.13 \AA$ and $d^{\prime \prime}(\mathrm{HF})_{\min } \sim 1.47$ $\AA$ is not populated (cf. Figures 5-7) and so $d, \rho_{c}$ pairs in the
vicinity of $d_{\text {sym }}$ and at $d_{\text {sym }}$ itself are not accessible. As discussed above, HF optimization of prosymmetric species never reached symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ geometries, and attempts to reduce the gap by goal-oriented manipulation of the chemistry of the nonprosymmetric species were not successful. The MP2 set, on the other hand, does contain species with optimized symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ geometries, but the density of $d, \rho_{\mathrm{c}}$ points in the $d_{\text {sym }}$ region is not sufficient to define the shape, on the two sides, of the convergence of the $d, \rho_{\mathrm{c}}$ regression function to $d_{\mathrm{sym}}$.

A more determined effort to expand the MP2 set, even if only by reoptimizing additional species of Table 1, may well help to elucidate the nature of the $\mathrm{N}-\mathrm{H}-\mathrm{N} d, \rho_{\mathrm{c}}$ correlation, but ultimately an (as yet unspecified) more realistic and comprehensive set of optimized ab initio geometries will be required to resolve the problems brought to light by the present investigation. The ab initio approach seems at present to show more immediate promise of success than reliance on an eventual accumulation of a sufficiently large, well-conditioned set of experimental results: even for $\mathrm{O}-\mathrm{H}-\mathrm{O}$ bonds, for which more abundant experimental results exist, extraction of reasonably accurate $d, \rho_{\mathrm{c}}$ values poses problems, especially at and in the vicinity of $d_{\text {sym }}$ (cf. above). ${ }^{13 \mathrm{~b}, 31}$ As well, it may not be possible to realize, experimentally, $\mathrm{N}-\mathrm{H}-\mathrm{N}$ containing systems expected to have very short, though not necessarily symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds, as such systems may have energetically and structurally more favorable chemical alternatives of no interest in the present context. This restrictive access to short $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds has implications for the $\rho_{\mathrm{c}}$ related quantities such as the Laplacian at the BCP; these will be discussed in a subsequent paper.

## Summary and Outlook

Analysis of the geometries of linear or near-linear $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bonds in 67 molecular species with geometries optimized at the RHF/6-31G** level (the HF set) and in 19 species optimized at the MP2/6-31G** level (the MP2 set) confirm that the $d^{\prime}=$ $\mathrm{N}-\mathrm{H}$ and $d^{\prime \prime}=\mathrm{H} \cdots \mathrm{N}$ distances in a bond are correlated (equations (1) $+(4)$ and (1) $+(4 a)$, respectively). The $d^{\prime}, d^{\prime \prime}$ correlation for the MP2 set is statistically equivalent to Steiner's regression $(1)+(2)$ based on the results of 19 neutron diffraction studies, which inspires confidence in the ab initio $d^{\prime}, d^{\prime \prime}$ values and in the model function. The correlation also makes it possible to estimate the position of the H atom in an $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bond from the $\mathrm{N} \cdots \mathrm{N}$ distances. This is of practical consequence to the experimentalist, since in structure determinations by X-ray diffraction the positions of the N atoms generally are reliably known, whereas that of the H atom may not be; in structure determinations by neutron diffraction the correlation can be used to verify the experimentally determined position of the proton. These applications are particularly important when the $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bond is short and close to symmetric. The question of whether all linear symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds have essentially the same dimensions, as predicted by the $d^{\prime}, d^{\prime \prime}$ correlation, is open to further enquiry by extending the existing MP2 and exploring post-MP2 calculations, and by making an effort to realize additional symmetric or near-symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$ bonds by experiment.

The positions of the bond critical points $x^{\prime}$ and $x^{\prime \prime}$ in an $\mathrm{N}-\mathrm{H} \cdots \mathrm{N}$ bond also are correlated. The correlation function resembles that for $d^{\prime} / d^{\prime \prime}$ (eqs (6) and (6a), and Figures 2B,D); furthermore, $x^{\prime}+x^{\prime \prime}=c_{0}+c_{1} D$.

No theoretically based functional dependence of the electron density $\rho_{\mathrm{c}}$ at the BCP appears to have been proposed, hence the uncertainty about the relative suitability of the individual model functions tried. While fitting the $d, \rho_{\mathrm{c}}$ pairs by a single
power (7) or exponential (8) function yields acceptable regressions for both the HF and the MP2 set, ${ }^{32}$ neither function accommodates the $d^{\prime}, \rho^{\prime}$ and $d^{\prime \prime}, \rho^{\prime \prime}$ as well as a combination of two functions of the same type, one for the $\mathrm{N}-\mathrm{H}$ and another for the $\mathrm{H} \cdots \mathrm{N}$ bonds, i.e., the variation of $\rho_{\mathrm{c}}$ with $d$ over the entire $d$ range appears to be continuous but not smooth (Figure 5). It remains to be seen whether this finding is universally valid and verifiable by ab initio calculations at levels beyond the scope of the present investigation as well as by experiment or whether it is specific to our particular data sets. Alternatively, it may be possible to construct a regression function that fits the $d, \rho_{\mathrm{c}}$ data satisfactorily and is continuous and smooth but incorporates terms additional to those in (7) and (8). This empirical possibility bears investigating in the future unless, or until, a proper theoretical basis for the $d, \rho_{\mathrm{c}}$ variation has been established.

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[^0]:    * To whom correspondence should be addressed. Fax: (902) 494-1310.
    † E-mail: russell.boyd@dal.ca.

[^1]:    ${ }^{a}$ NHN bond angle, total electronic energy $E$ (au), energy of formation $E_{\mathrm{f}}$ from component parts (au), net atom charges $\epsilon$ (Mulliken, e), and other point-group symmetries in which optimization was attempted. Arranged in the order of increasing $D .{ }^{b}$ Optimized configuration of lowest energy. 8: staggered configuration, $\mathrm{NHN}=174.6^{\circ}$ is the angle inside the planar $\mathrm{FN}_{\mathrm{d}} \mathrm{H}^{\cdots} \mathrm{N}_{\mathrm{a}}$ arc. 65 , anti. ${ }^{c}$ Point-group symmetry. Where proper optimization (no negative frequencies) could not be achieved, the lowest-energy optimization is listed and the corresponding PG symbol is in brackets. ${ }^{d}\left(180^{\circ}\right)$ indicates that the angle is $180^{\circ}$ by symmetry. ${ }^{e} E_{\mathrm{f}}=E$ (product) $-\sum E$ (component parts). The two component parts are indicated by the dotted bond and are of matching symmetry. ${ }^{f}[\mathrm{PG}]$ indicates failure to optimized properly in that PG. The other PGs listed resulted in higher energies than that reported in the table. ${ }^{g}$ Ring structure with the H -bonds in the ring: 20, $\mathbf{F N}-\mathbf{H} \cdots \cdots \mathbf{N}$, bifurcated $\mathrm{F} \cdots 2 \mathrm{H}$ bond;
    
    $\mathbf{N}-\mathrm{C}-\mathrm{S}$.
    $\mathbf{H}-\mathrm{N}\left(\mathrm{H}_{2}\right)-\mathrm{H}$

[^2]:    ${ }^{a}$ Direct regression, not via the logarithmically transformed function. ${ }^{b}$ sym indicates symmetric $\mathrm{N}-\mathrm{H}-\mathrm{N}$. ${ }^{c} \sigma$ as percentage of the respective $\rho_{c}$

